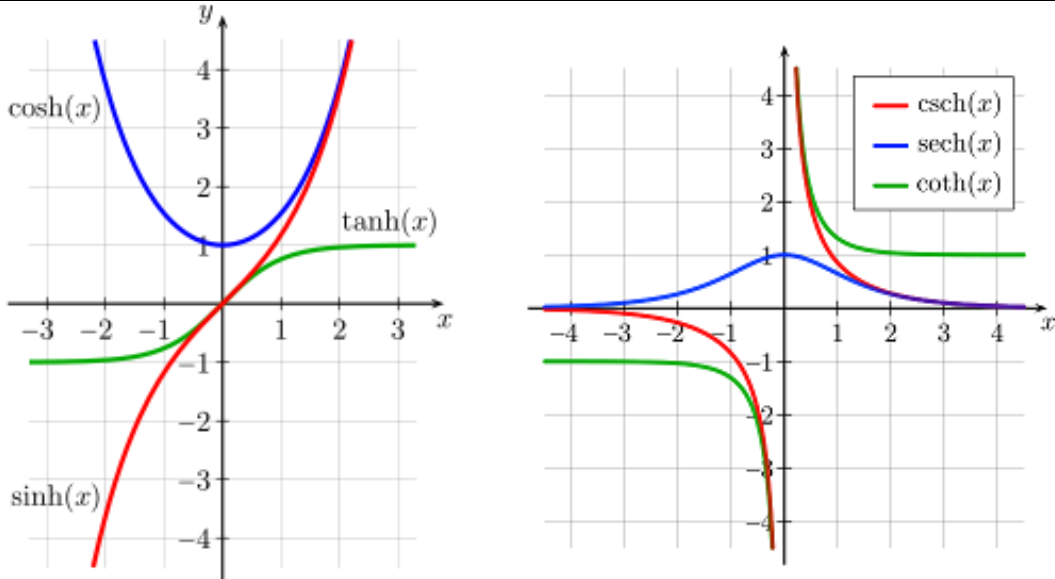


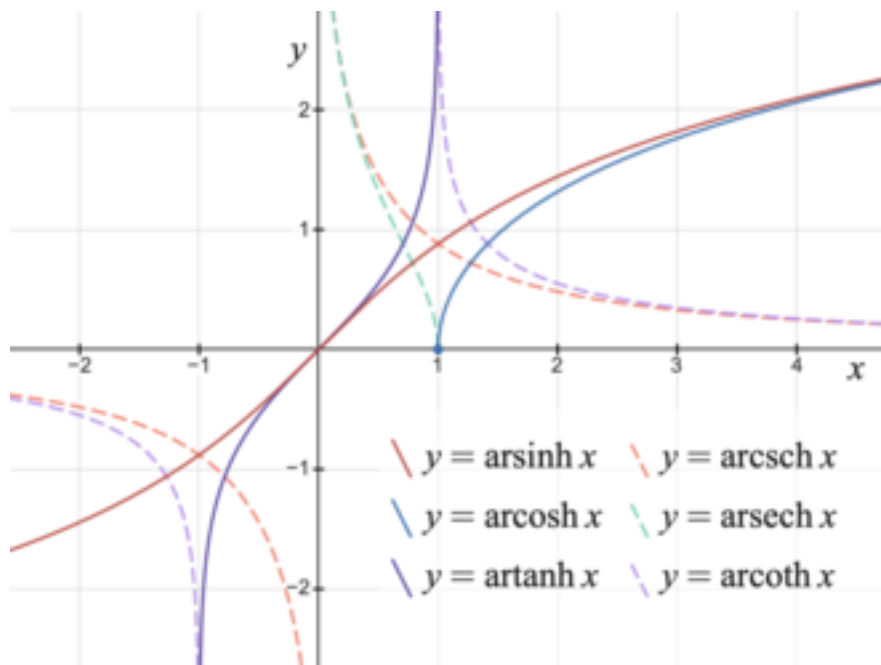
# Harold's Hyperbolics Cheat Sheet

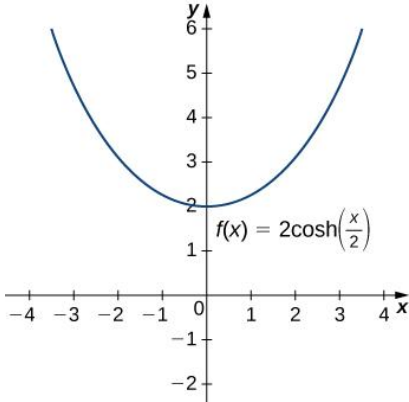
7 January 2026

## Hyperbolic Graphs

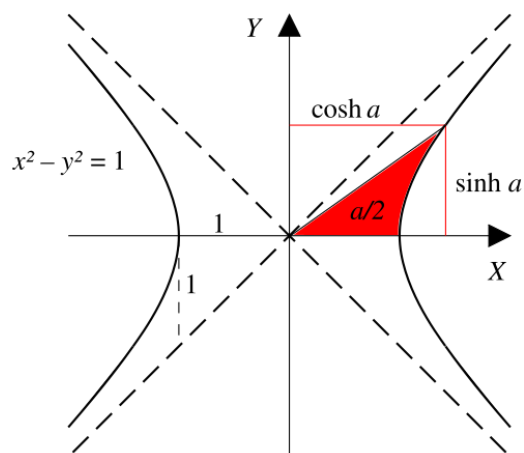


## Inverse Hyperbolic Graphs



Hyperbolic Definitions		<i>sinh &amp; cosh</i>	$e^x$	Complex
1. Hyperbolic Sine	$\sinh(x)$	$\frac{1}{\cosh(x)}$	$\frac{e^x - e^{-x}}{2}$	$-i \sin(ix)$
2. Hyperbolic Cosine (catenary)	$\cosh(x)$	$\frac{1}{\sinh(x)}$	$\frac{e^x + e^{-x}}{2}$	$\cos(ix)$
3. Hyperbolic Tangent	$\tanh(x)$	$\frac{\sinh(x)}{\cosh(x)}$	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	$-i \tan(ix)$
4. Hyperbolic Cotangent	$\coth(x)$	$\frac{\cosh(x)}{\sinh(x)}$	$\frac{e^x + e^{-x}}{e^x - e^{-x}}$	$i \cot(ix)$
5. Hyperbolic Secant	$\operatorname{sech}(x)$	$\frac{1}{\cosh(x)}$	$\frac{2}{e^x + e^{-x}}$	$\sec(ix)$
6. Hyperbolic Cosecant	$\operatorname{csch}(x)$	$\frac{1}{\sinh(x)}$	$\frac{2}{e^x - e^{-x}}$	$i \csc(ix)$
7. Inverse Hyperbolic Sine	$\sinh^{-1}(x)$	$\ln(x + \sqrt{x^2 + 1})$		
8. Inverse Hyperbolic Cosine	$\cosh^{-1}(x)$	$\ln(x + \sqrt{x^2 - 1}), \quad x \geq 1$		
9. Inverse Hyperbolic Tangent	$\tanh^{-1}(x)$	$\frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), \quad  x  < 1$		
10. Inverse Hyperbolic Cotangent	$\coth^{-1}(x)$	$\frac{1}{2} \ln\left(\frac{x+1}{x-1}\right), \quad  x  > 1$		
11. Inverse Hyperbolic Secant	$\operatorname{sech}^{-1}(x)$	$\ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right), \quad 0 < x \leq 1$ $\ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right), \quad 0 < x \leq 1$		
12. Inverse Hyperbolic Cosecant	$\operatorname{csch}^{-1}(x)$	$\ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right), \quad x \neq 0$		
Catenary	$y = a \cosh\left(\frac{x}{a}\right)$			

Hyperbolic Identities	
Odd and Even	$\sinh(-x) = -\sinh(x)$
	$\cosh(-x) = \cosh(x)$
	$\tanh(-x) = -\tanh(x)$
Pythagorean Identities	<b><math>\cosh^2(x) - \sinh^2(x) = 1</math></b> (Euler's formula)
	$\operatorname{sech}^2(x) = 1 - \tanh^2(x)$
	$\operatorname{csch}^2(x) = \operatorname{coth}^2(x) - 1$
Sums/Difference of Angles	$\sinh(x \pm y) = \sinh(x) \cosh(y) \pm \cosh(x) \sinh(y)$
	$\cosh(x \pm y) = \cosh(x) \cosh(y) \pm \sinh(x) \sinh(y)$
	$\tanh(x \pm y) = \frac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x) \tanh(y)}$
Double Angle	$\sinh(2x) = 2 \sinh(x) \cosh(x)$
	$\cosh(2x) = \sinh^2(x) + \cosh^2(x)$ $= 2 \cosh^2(x) - 1$ $= 1 + 2 \sinh^2(x)$
	$\tanh(2x) = \frac{2 \tanh(x)}{1 + \tanh^2(x)}$
Half Angle	$\sinh\left(\frac{x}{2}\right) = \frac{\sinh(x)}{\sqrt{2(\cosh(x) + 1)}}$
	$\cosh\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh(x) + 1}{2}}$
	$\tanh\left(\frac{x}{2}\right) = \frac{\sinh(x)}{\cosh(x) + 1}$ $= \frac{\cosh(x) - 1}{\sinh(x)}$ $= \operatorname{coth}(x) - \operatorname{csch}(x)$ if $x \neq 0$
Squared	$\sinh^2(x) = \frac{1}{2}(\cosh(2x) - 1)$
	$\cosh^2(x) = \frac{1}{2}(\cosh(2x) + 1)$
Exponents (Powers)	$(\cosh(x) + \sinh(x))^n = \cosh(nx) + \sinh(nx)$
Natural Exponential	$e^x = \cosh(x) + \sinh(x)$
	$e^{-x} = \cosh(x) - \sinh(x)$
Natural Logarithmic	$\ln(x) = \pm \cosh^{-1}\left(\frac{x^2 + 1}{2x}\right)$ $= \sinh^{-1}\left(\frac{x^2 - 1}{2x}\right)$ $= \tanh^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right)$



Hyperbolic Derivatives	
1. Hyperbolic Sine	$\frac{d}{dx} [\sinh(x)] = \cosh(x)$
2. Hyperbolic Cosine	$\frac{d}{dx} [\cosh(x)] = \sinh(x)$
3. Hyperbolic Tangent	$\frac{d}{dx} [\tanh(x)] = \operatorname{sech}^2(x)$
4. Hyperbolic Cotangent	$\frac{d}{dx} [\coth(x)] = -\operatorname{csch}^2(x), \quad x \neq 0$
5. Hyperbolic Secant	$\frac{d}{dx} [\operatorname{sech}(x)] = -\operatorname{sech}(x) \tanh(x)$
6. Hyperbolic Cosecant	$\frac{d}{dx} [\operatorname{csch}(x)] = -\operatorname{csch}(x) \coth(x), \quad x \neq 0$
7. Hyperbolic Arcsine	$\frac{d}{dx} [\sinh^{-1}(x)] = \frac{1}{\sqrt{x^2 + 1}}$
8. Hyperbolic Arccosine	$\frac{d}{dx} [\cosh^{-1}(x)] = \frac{1}{\sqrt{x^2 - 1}}, \quad x > 1$
9. Hyperbolic Arctangent	$\frac{d}{dx} [\tanh^{-1}(x)] = \frac{1}{1 - x^2}, \quad  x  < 1$
10. Hyperbolic Arccotangent	$\frac{d}{dx} [\coth^{-1}(x)] = \frac{1}{1 - x^2}, \quad  x  > 1$
11. Hyperbolic Arcsecant	$\frac{d}{dx} [\operatorname{sech}^{-1}(x)] = \frac{-1}{x\sqrt{1 - x^2}}, \quad 0 < x < 1$
12. Hyperbolic Arccosecant	$\frac{d}{dx} [\operatorname{csch}^{-1}(x)] = \frac{-1}{ x \sqrt{1 + x^2}}, \quad x \neq 0$

Hyperbolic 2 <sup>nd</sup> Derivatives	
1. Hyperbolic Sine	$\frac{d^2}{dx^2} [\sinh(x)] = \sinh(x)$
2. Hyperbolic Cosine	$\frac{d^2}{dx^2} [\cosh(x)] = \cosh(x)$

Hyperbolic Integrals	(See: <a href="#">Wikipedia, list of integrals of hyperbolic functions.</a> )
1. Hyperbolic Sine	$\int \sinh(x) dx = \cosh(x) + C$
2. Hyperbolic Cosine	$\int \cosh(x) dx = \sinh(x) + C$
3. Hyperbolic Tangent	$\int \tanh(x) dx = \ln \cosh(x)  + C$
4. Hyperbolic Cotangent	$\int \coth(x) dx = \ln \sinh(x)  + C$
5. Hyperbolic Secant	$\int \operatorname{sech}(x) dx = \ln \operatorname{sech}(x) + \tanh(x) $ $= \tanh^{-1}(\sinh(x)) + C$ $= 2 \tan^{-1} \left  \tanh\left(\frac{x}{2}\right) \right  + C$
6. Hyperbolic Cosecant	$\int \operatorname{csch}(x) dx = \ln \coth(x) - \operatorname{csch}(x)  + C$ $= \coth^{-1}(\cosh(x)) + C$ $= \ln \left  \tanh\left(\frac{x}{2}\right) \right $
7. Hyperbolic Secant <sup>2</sup>	$\int \operatorname{sech}^2(x) dx = \tanh(x) + C$
8. Hyperbolic Cosecant <sup>2</sup>	$\int \operatorname{cosh}^2(x) dx = -\coth(x) + C$
9. Hyperbolic Arcsine	$\int \sinh^{-1}\left(\frac{x}{c}\right) dx = x \sinh^{-1}\left(\frac{x}{c}\right) - \sqrt{x^2 + c^2} + C$
10. Hyperbolic Arccosine	$\int \cosh^{-1}\left(\frac{x}{c}\right) dx = x \cosh^{-1}\left(\frac{x}{c}\right) - \sqrt{x^2 - c^2} + C$
11. Hyperbolic Arctangent	$\int \tanh^{-1}\left(\frac{x}{c}\right) dx = x \tanh^{-1}\left(\frac{x}{c}\right) + \frac{c}{2} \ln c^2 - x^2  + C,$ $ x  <  c $
12. Hyperbolic Arccotangent	$\int \coth^{-1}\left(\frac{x}{c}\right) dx = x \coth^{-1}\left(\frac{x}{c}\right) + \frac{c}{2} \ln x^2 - c^2  + C,$ $ x  >  c $
13. Hyperbolic Arcsecant	$\int \operatorname{sech}^{-1}\left(\frac{x}{c}\right) dx = x \operatorname{sech}^{-1}\left(\frac{x}{c}\right) - c \tan^{-1} \frac{x \sqrt{\frac{c-x}{c+x}}}{x-c} + C,$ $0 < x < c$
14. Hyperbolic Arccosecant	$\int \operatorname{csch}^{-1}\left(\frac{x}{c}\right) dx = x \operatorname{csch}^{-1}\left(\frac{x}{c}\right) + c \ln \frac{x + \sqrt{x^2 + c^2}}{c} + C, \quad 0$ $< x < c$
9. Hyperbolic Arcsine	$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$
10. Hyperbolic Arccosine	$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left \frac{x}{a}\right  + C$

<b>11. Hyperbolic Arctangent</b>	$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + C, \quad x^2 < a^2$
<b>12. Hyperbolic Arccotangent</b>	$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right) + C, \quad x^2 > a^2$
<b>13. Hyperbolic Arcsecant</b>	$\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \operatorname{sech}^{-1}\left \frac{x}{a}\right  + C$
<b>14. Hyperbolic Arccosecant</b>	$\int \frac{1}{x\sqrt{a^2 + x^2}} dx = -\frac{1}{a} \operatorname{csch}^{-1}\left \frac{x}{a}\right  + C$

### Sources

- Wikipedia (2025). Hyperbolic Functions. [https://en.wikipedia.org/wiki/Hyperbolic\\_functions](https://en.wikipedia.org/wiki/Hyperbolic_functions).

### See Also

- [Harold's Trigonometry & Hyperbolic Parent Functions Cheat Sheet](#)