Harold's Philosophical Logic
Cheat Sheet

1 December 2025

The 7 Basic Logical Symbols

Operator	Symbol	Example	English Example
1) Intersection (AND)	∩, •	A • B	• Conjunction • A and B • A, but B • despite the fact that A, B • even though A, B • although A, B • overlap
2) Union (OR)	∪, ∨	A ∨ B	• Disjunction • A or B • inclusive or • both combined
3) Negation (NOT)	~, $\bar{A}$	~A	• not A
4) Conditional	→, ⊃	A ⊃ B	• if A then q • if A, B • B if A • A implies B • A only if B • B in case that A • A is sufficient for B • B is necessary for A
5) Biconditional	↔, ⟷, 🡘, ⇔, ⟺	p ⟷ q	• A iff B • A if and only if B • A is necessary and sufficient for B • if A then B, and conversely • if not A then not B, and conversely
6) Universal Quantifier	(*x*), ∀*x*	(*x*) *p*(*x*)	• for all • for any • for each
7) Existential Quantifier	(∃*x*)	(∃*x*) *p*(*x*)	• there exists • there is at least one
Equivalence (See Biconditional)	≡	expression$_1$ ≡ expression$_2$	• is identical to • is equivalent to • is defined as • the two expressions always have the same truth value

"… the structure of all mathematical statements can be understood using these symbols, and all mathematical reasoning can be analyzed in terms of the proper use of these symbols."
Source: "How to Prove It: A Structured Approach", 3rd Edition, p. 75.

Logical Truth Tables

A	B	AND •	NOT AND ~•	OR ∨	NOT OR ~∨	XOR ⊻, ⊕	NOT XOR ⊙	NOT ~ (~A)	If ... Then ⊃	Iff ≡	Taut-ology (True) T	Contra-diction (False) F
F	F	F	T	F	T	F	T	T	T	T	T	F
F	T	F	T	T	F	T	F	T	T	F	T	F
T	F	F	T	T	F	T	F	F	F	F	T	F
T	T	T	F	T	F	F	T	f	T	T	T	F

Blank Truth Tables

Inputs				Output		
A	B	C	D	X	Y	Z
F	F	F	F			
F	F	F	T			
F	F	T	F			
F	F	T	T			
F	T	F	F			
F	T	F	T			
F	T	T	F			
F	T	T	T			
T	F	F	F			
T	F	F	T			
T	F	T	F			
T	F	T	T			
T	T	F	F			
T	T	F	T			
T	T	T	F			
T	T	T	T			

Inputs			Output	
A	B	C	X	Y
F	F	F		
F	F	T		
F	T	F		
F	T	T		
T	F	F		
T	F	T		
T	T	F		
T	T	T		

Inputs		Output
A	B	X
F	F	
F	T	
T	F	
T	T	

Precedence Rules (PEMDAS for Logic)

#	Operator	Symbol	Precedence
1	Parenthesis	( )	Highest precedence
2	Logical NOT	~	
3	Quantifiers	(x), (∃x)	
4	Logical AND	•	Applied Left to Right
5	Logical OR	∨	
6	Logical Conditional	⊃	
7	Logical Biconditional	≡	Lowest precedence

Logical Conditional Connective Laws

Law or Statement	Logical Expression	Is Equivalent To (≡)		Description
Antecedent / Consequent	If <Antecedent> then <Consequent> . <Consequent> if <Antecedent> .			The Antecedent immediately follows the “if” statement.
Conditional Laws	p ⊃ q	≡	~p ∨ q ~(p • ~q) Logical Equivalences: p ∨ q ≡ ~p ⊃ q p • q ≡ ~(p ⊃ ~q) ~(p ⊃ q) ≡ p • ~q (p ⊃ q) • (p ⊃ r) ≡ p ⊃ (q • r) (p ⊃ q) ∨ (p ⊃ r) ≡ p ⊃ (q ∨ r) (p ⊃ r) • (q ⊃ r) ≡ (p • q) ⊃ r (p ⊃ r) ∨ (q ⊃ r) ≡ (p ∨ q) ⊃ r	Conditional, If ... Then, Implication
Biconditional Laws (Equivalence)	p ≡ q p ↔ q	≡	(p ⊃ q) • (q ⊃ p) (p ⊃ q) • (~p ⊃ ~q) (p • q) ∨ (~p • ~q) ~p ↔ ~q Logical Equivalences: ~ (p ↔ q) ≡ p ↔ ~q	Bi-conditional, If and only If, iff, XNOR Is equivalent to
Converse*	p ⊃ q	≢	q ⊃ p	False
Inverse*	p ⊃ q	≢	~p ⊃ ~q	False

Rules of Implication

(Inference with Propositions)

Rule Name	Rule Logic	English Example
Hypothesis	Givens. First lines of a proof.	It is raining today. You live in McKinney, Texas.
Therefore	$\therefore$	Therefore. In conclusion.
1) Modus Ponens (MP)	p $\underline{p \supset q}$ $\therefore q$	It is raining today. If it is raining today, I will not ride my bike to school. Therefore, I will not ride my bike to school.
2) Modus Tollens (MT)	$\sim q$ $\underline{p \supset q}$ $\therefore \sim p$	If Sam studied for his test, then Sam passed his test. Sam did not pass his test. Therefore, Sam did not study for his test.
3) Hypothetical Syllogism (HS) (Transitivity)	$p \supset q$ $\underline{q \supset r}$ $\therefore p \supset r$	If you are mad, then you will yell. If you yell, then you will wake the baby. Therefore, if you are mad, then you will wake the baby.
4) Disjunctive Syllogism (DS) (Elimination)	$p \vee q$ $\underline{\sim p}$ $\therefore q$	Sam studied for his test or Sam took a nap. Sam did not study for his test. Therefore, Sam took a nap.
5) Constructive Dilemma (CD)	$p \vee q$ $\underline{(p \supset r) \bullet (q \supset s)}$ $\therefore r \vee s$	Oscar is either a dog or a cat. If Oscar is a dog, then you'll have fleas, and if Oscar is a cat, then you'll have fur balls. Therefore, you'll have either fleas or fur balls.
6) Simplification (Simp) (Specialization)	$\underline{p \bullet q}$ $\therefore p$	It is rainy today and it is windy today. Therefore, it is rainy today.
7) Conjunction (Conj)	p $\underline{q}$ $\therefore p \bullet q$	Sam studied for his test. Sam passed his test. Sam studied for his test and passed his test.
8) Addition (Add) (Generalization)	$\underline{p}$ $\therefore p \vee q$	It is raining today. Therefore, it is either raining today or snowing today or both.
Resolution	$p \vee q$ $\underline{\sim p \vee q}$ $\therefore q \vee r$	Your shirt is red or your pants are blue. Your shirt is not red or your pants are blue. Therefore, your pants are blue or your shoes are white.

Proof by Division into Cases	$p \lor q$ $p \supset r$ $q \supset r$ $\therefore r$	It is raining or it is Monday. It is raining, so it is wet. It is Monday, so it is wet. It is wet.
Contradiction Rule	$\sim p \supset F$ $\therefore p$	If it is not raining is a false statement; then it is raining.

Rules of Replacement

(Logical Connective Laws / Equivalences / Inference)

Law	“Or” Example	“And” Example
9. De Morgan’s rule (DM) (Propositional Logic)	p ∨ q ≡ ~(~p • ~q) ~(p ∨ q) ≡ ~p • ~q (p ∨ ~q) ⊃ r ≡ ~r ⊃ (~p • q)	p • q ≡ ~ (~p ∨ ~q) ~(p • q) ≡ ~p ∨ ~q (p • ~q) ⊃ r ≡ ~r ⊃ (~p ∨ q)
10. Commutative (Com)	p ∨ q ≡ q ∨ p	p • q ≡ q • p
11. Associative (Assoc)	(p ∨ q) ∨ r ≡ p ∨ (q ∨ r)	(p • q) • r ≡ p • (q • r)
12. Distributive (Dist)	p • (q ∨ r) ≡ (p • q) ∨ (p • r)	p ∨ (q • r) ≡ (p ∨ q) • (p ∨ r)
13. Double Negations (DN) (Involution Law)	~ ~p ≡ p	
14. Transposition (Trans) (Contrapositive)	(p ⊃ q) ≡ (~q ⊃ ~p)	
15. Material Implication (Impl)	(p ⊃ q) ≡ (~p ∨ q)	
16. Material Equivalence (Equiv)	(p ≡ q) ≡ [(p • q) ∨ (~p • ~q)]	(p ≡ q) ≡ [(p ⊃ q) • (q ⊃ p)]
17. Exportation (Exp)	[(p ∨ q) ⊃ r] ≡ [(p ⊃ r) ∨ (q ⊃ r)]	[(p • q) ⊃ r] ≡ [p ⊃ (q ⊃ r)]
18. Tautology (Taut) (Idempotent)	p ≡ (p ∨ p)	p ≡ (p • p)
Contradiction (Identity)	p ∨ F ≡ p	p • T ≡ p
Domination or Null (Universal Bound Laws)	p ∨ T ≡ T	p • F ≡ F
Negation or Complement (Complementary Laws)	p ∨ ~p ≡ T ~F ≡ T	p • ~p ≡ F ~T ≡ F
Uniting	(p • q) ∨ (p • ~q) ≡ p	(p ∨ q) • (p ∨ ~q) ≡ p
Absorption	p ∨ (p • q) ≡ p	p • (p ∨ q) ≡ p
Multiplying and Factoring Laws	(p ∨ q) • (~p ∨ r) ≡ (p • r) ∨ (~p • q)	(p • q) ∨ (~p • r) ≡ (p ∨ r) • (~p ∨ q)
Consensus	(p • q) ∨ (q • r) ∨ (~p • r) ≡ (p • q) ∨ (~p • r)	(p ∨ q) • (q ∨ r) • (~p ∨ r) ≡ (p ∨ q) • (~p ∨ r)
Exclusive Or (⊕)	p ⊕ q ≡ (p ∨ q) ∨ ~(p • q)	p ⊕ q ≡ (~p • q) ∨ (p ∨ ~q)

Proof Methods

<table>
<tr><th>Method</th><th>Definition</th></tr>
<tr><td>Direct (DP)</td><td>• Assume the hypothesis is true, then use logical steps to arrive at the conclusion.
• Assume <i>p</i> is true, then conclude <i>q</i>.</td></tr>
<tr><td>Indirect (IP)
(Contradiction)</td><td>• Assume the opposite of what you want to prove, then show this leads to a contradiction.
• To prove p, assume $\sim p$ and derive a contradiction, such as $\sim q \bullet q$.
• If some statement is assumed true, and a logical contradiction occurs, then the statement must be false.
• Can also be a proof by counterexample.
  ○ E.g., Assume ~(p ⊃ q), which is equivalent to p • ~q.
• Assumption for Indirect Proof (AIP)</td></tr>
<tr><td>Conditional (CP)</td><td>• Assume a hypothesis <u>temporarily</u> is true to derive a conclusion.
• Assume p, derive q; conclude $p \supset q$.
• The goal is not to prove <i>p</i> is true in reality, but to prove that if <i>p</i> were true, then <i>q</i> would necessarily follow.
• Assumption for Conditional Proof (ACP)</td></tr>
<tr><td>Contrapositive</td><td>• Modus Tollens.
• Infers the statement p ⊃ <i>q</i> by establishing the logically equivalent contrapositive statement: ~q ⊃ ~p.
• When given p ⊃ <i>q</i>, assume ~q is true, then prove ~p.
• We prove that if the negation of the original conclusion is false, then the negation of the initial theorem is false.
• Relies on De Morgen's Law.
<table>
<tr><th>p</th><th>q</th><th>p ⊃ q</th><th>Technique</th></tr>
<tr><td>F</td><td>F</td><td>T</td><td>Modus Tollens</td></tr>
<tr><td>F</td><td>T</td><td>T</td><td>(seems forced)</td></tr>
<tr><td>T</td><td>F</td><td>F</td><td></td></tr>
<tr><td>T</td><td>T</td><td>T</td><td>Modus Ponens</td></tr>
</table>
• A proof by contrapositive is a special case of a proof by contradiction (indirect).</td></tr>
<tr><td>Construction
(Example)</td><td>• The construction of a concrete example with a property to show that something having that property exists.
• AKA proof by example.</td></tr>
<tr><td>Exhaustion / By Cases</td><td>• The conclusion is established by dividing it into a finite number of cases and proving each one separately.</td></tr>
<tr><td>Induction</td><td>• A single "base case" is proved, and an "induction rule" is proved that establishes that any arbitrary case implies the next case.</td></tr>
</table>

Proof Examples with Solutions

Indirect Proof	Logical Solution Steps	Rules
Given: 1. (N ∨ O) ⊃ (C • D) 2. (D ∨ K) ⊃ (P ∨ ~C) 3. (P ∨ G) ⊃ ~(N • D) **Prove:** ~N	1. (N ∨ O) ⊃ (C • D)	Given
	2. (D ∨ K) ⊃ (P ∨ ~C)	Given
	3. (P ∨ G) ⊃ ~(N • D)	Given
	4. N	AIP
	5. N ∨ O	4, Add
	6. C • D	1, 5, MP
	7. D • C	6, Com
	8. D	7, Simp
	9. D ∨ K	8, Add
	10. P ∨ ~C	2, 9, MP
	11. C	6, Simp
	12. ~C ∨ P	10, Com
	13. ~~C	11, DN
	14. P	12, 13, DS
	15. P ∨ G	14, Add
	16. ~(N • D)	3, 15, MP
	17. ~N ∨ ~D	16, DM
	18. ~~N	4, DN
	19. ~D	17, 18, DS
	20. D • ~D	8, 19, Conj
	21. ~N	4-20, IP

Conditional Proof	Logical Solution Steps	Rules
Given: 1. Q ⊃ (R ⊃ S) 2. Q ⊃ (T ⊃ ~U) 3. U ⊃ (R ∨ T) **Prove:** Q ⊃ (U ⊃ S)	1. Q ⊃ (R ⊃ S)	Given
	2. Q ⊃ (T ⊃ ~U)	Given
	3. U ⊃ (R ∨ T)	Given
	4. Q	ACP
	5. U	ACP
	6. R ⊃ S	1, 4 MP
	7. T ⊃ ~U	2, 4, MP
	8. ~~U	5, DN
	9. ~T	7, 8, MT
	10. R ∨ T	3, 5, MP
	11. T ∨ R	10, Com
	12. R	9, 11, DS
	13. S	6, 12, MP
	14. U ⊃ S	5-13, CP
	15. Q ⊃ (U ⊃ S)	4-14, CP

Logical Quantifiers

Definition	Logical Expression	Is Equivalent To (≡)	English Example
Universal Quantifier (x)	$(x)\ Px$ $(x) \in Px$ $(x) \in \mathbb{D},\ Px$ (x), *if x is in* $\mathbb{D}$ *then* Px	"For all *x* in the domain, Px is true" $(x) \in A\ Px \equiv (x)\ (x \in A \supset Px)$ For the finite set domain of discourse $\{a_1, a_2, \ldots, a_k\}$, $(x)\ Px \equiv Pa_1 \bullet Pa_2 \bullet \ldots \bullet Pa_k$	• for all • all elements • for each member • any • anyone • anything • every • everyone • everybody • everything • x could be anything at all • whoever • none but • only <plural noun> • the only (pronoun) • no … except <plural noun>
Existential Quantifier (∃x)	$(\exists x)\ Px$ $(\exists x) \in Px$ $(\exists x) \in \mathbb{D},\ Px$	"There exists x in the domain, such that Px is true" For the finite set domain of discourse $\{a_1, a_2, \ldots, a_k\}$, $(\exists x)\ Px \equiv Pa_1 \vee Pa_2 \vee \ldots \vee Pa_k$ $Px \neq \emptyset$	• there exists an x • there is • some • someone • somebody • something • at least one value of x • there is at least one x • it is the case that • the truth set is not equal to ∅ • a few
Uniqueness Quantifier (∃!)	$\exists! x\ Px$	there is a unique x in Px such that … $(\exists x)\ (Px \bullet {\sim}(y)\ (Py \bullet y \neq x))$ $(\exists x)\ (Px \bullet (y)\ (Py \supset y = x))$ $(\exists x)\ (y)\ (Py \equiv y = x)$ $(\exists x)\ Px \bullet (y)\ (z)\ ((Py \bullet Pz) \supset y = z)$	• unique • there is a unique x • there exists exactly one • there is exactly one x such that *P(x)*
Negated Existential Quantifier	$\sim [(\exists x)\ Px]$	$(x)\ {\sim}Px$	• nobody • no one • not one • there does not exist
	$\sim [(x)\ Px]$	$(\exists x)\ {\sim}Px$	

Quantifier Logic Examples

Quantifier	Symbolic Translations	English Example
Everyone	(x) Px	• Everyone is <something>.
	(x) (y) Pxy NOTE: includes (x = y)	• Everyone <did something> to everyone.
Everyone Else	(x) (x ≠ y ⊃ Px)	• Everyone except <someone> is <something>.
	(x) (y) (*x* ≠ *y*) ⊃ Pxy NOTE: excludes (x = y)	• Everyone <did something> to everyone else.
Not Every	~(x) (Ax ⊃ Bx)	• It is not the case that every <something> <did something>.
	(∃x) ~Px	• Not every <something> <did something>.
Someone Else	(∃x) (x ≠ y ∨ Px)	• Someone other than <someone> is <something>.
	(x) (∃y) ((*x* ≠ *y*) • Pxy) NOTE: excludes (x = y)	• Everyone <did something> to <someone> else.
Exactly One	∃!*x* Px	• Exactly one person <did something>.
	(∃x) (Px • (y) ((*x* ≠ *y*) ⊃ ~Py)) ≡	
No One	~(∃x) Px	• No one <did something>.
	(x) ~Px	

Rules of Inference with Quantifiers

Rule Name	Rule Logic	English Example
Variables	**x** : Quantified variable	The domain is the set of all integers.
Elements	**c, d** : Elements of the domain, arbitrary or particular	c is a particular integer. Element definition.
Universal Instantiation (UI)	c is an element (arbitrary or particular) $(x)\ Px$ $\therefore\ Pc$	Sam is a student in the class. Every student in the class completed the assignment. Therefore, Sam completed his assignment.
Universal Generalization (UG)	c is an arbitrary element Pc $\therefore\ (x)\ Px$	All psychiatrists are doctors. All doctors are college graduates. Therefore, all psychiatrists are college graduates. 1. $(x)\ (Px \supset Dx)$ Given 2. $(x)\ (Dx \supset Cx)$ Given 3. $Px \supset Dx$ 1, UI 4. $Dx \supset Cx$ 2, UI 5. … 6. $Px \supset Cx$ 7. $\therefore\ (x)\ (Px \supset Cx)$ 6, UG
Existential Instantiation (EI)	$(\exists x)\ Px$ $\therefore$ (c is a particular element) • Pc	All attorneys are college graduates. Some attorneys are golfers. Therefore, some golfers are college graduates. i.e., If an object is known to exist, then that object can be given a name.
Existential Generalization (EG)	c is an element (arbitrary or particular) Pc $\therefore\ (\exists x)\ Px$	All tenors are singers. Andrea Bocelli is a tenor. Therefore, there is at least one singer. 1. $(x)\ (Tx \supset Sx)$ Given 2. Ta Given 3. $Ta \supset Sa$ 1, UI 4. Sa 2,3, MP 5. $\therefore\ (\exists x)\ Sx$ 4, EG

Quantifier Translation Hints

Statement Form	Symbolic Translation	English Example
A B	Ax • Bx	Pretty girl.
Either A or B	Ax ∨ Bx	Rachel is a journalist or a newscaster.
	(Ax ∨ ~Bx) ∨ (~Ax ∨ Bx)	Rachel is either a journalist or a newscaster, but not both. (XOR)
Neither A nor B	~Ax • ~Bx	Neither Wordsworth nor Shelley was Irish.
A is/are B	Ax ⊃ Bx	Eli is a student. Sea lions are mammals.
Anything is A	(x) Ax	Anything is conceivable.
All A are B	(x) (Ax ⊃ Bx)	All maples are trees.
Some A are B	(∃x) (Ax • Bx)	Some grapes are sour.
Some A are not B	(∃x) (Ax • ~Bx)	Some grapes are not sour.
A exist	(∃x) Ax	Tigers exist.
A do not exist	~(∃x) Ax (x) ~Ax	Unicorns do not exist.
No A are B	~(x) (Ax ⊃ Bx) (∃x) (Ax • ~Bx)	No novels are biographies.
Not a single A did B	(x) (Ax ⊃ ~Bx) ~(∃x) (Ax • Bx)	Not a single psychologist attended the convention.
Whoever is A is B	(x) (Ax ⊃ Bx)	Whoever is a socialite is vain.
Some A B iff C	(∃x) (Ax • (Bx ≡ Cx))	Some dogs bite if and only if they are teased.
Some A B are C	(∃x) [(Ax • Bx) • Cx]	Some French restaurants are exclusive.
A B are C	(x) [(Ax • Bx) ⊃ Cx]	Ripe apples are delicious.
A and B are C D	(x) [(Ax ∨ Bx) ⊃ (Cx • Dx)]	Violins and cellos are stringed instruments.
Only i is F.	Fi • (x) [Fx ⊃ (x = i)]	Only Sally is running.
The only F that is G is i.	Fi • Gi • (x) [(Fx • Gx) ⊃ (x = i)]	The only instrument that is brass is the trumpet.
No F except i is G.		No plants except for Venus flytraps are carnivorous.
All F except i are G.	Fi • ~Gi • (x) [(Fx • (x ≠ i)) ⊃ Gx]	All students except Billy are on time.
i is the F that is most so-and-so.	Fi • (x) [(Fx • (x ≠ i)) ⊃ i is more so-and-so than x]	Rex is the dog that is most loved than the rest.
There is at most one F.	(x) (y) [(Fx • Fy) ⊃ (x = y)]	There is at most one moon.
There are at least two F's.	(∃x) (∃y) [Fx • Fy • (x ≠ y)]	There are at least two moons.
There are exactly two F's.	(∃x) (∃y) { Fx • Fy • (x ≠ y) • (z) [Fz ⊃ ((z = x) v (z = y))] }	There are exactly two moons.
The F is G.	(∃x) [Fx • (y) (Fy ⊃ (y = x)) • Gx]	The moon is bright.

Quantifier Laws

Definition	Logical Expression	Is Equivalent To (≡)		English Example
Abbreviation	(∃x) (x ∈ A • ~Px)	≡	(∃x) ∈ A ~Px	Simplification
Expanding Abbreviation	(x) ∈ A Px	≡	(x) (x ∈ A ⊃ Px)	Complication
1. Quantifier Negation Laws (QN)	(x) Px	≡	~(∃x) ~Px	• everyone is perfect • no one is imperfect
	(∃x) Px	≡	~(x) ~Px	• somebody is perfect • nobody is imperfect
	~(x) Px	≡	(∃x) ~Px	• not everyone is perfect • someone is imperfect
	~(∃x) Px	≡	(x) ~Px	• nobody is perfect • everybody is imperfect
2. Conditional Law (ACP)	x ∈ A ⊃ Px	≡	x ∉ A ∨ Px	p ⊃ q ≡ ~p ∨ q
3. Subset Negation Law	x ∈ A	≡	~(x ∉ A)	Negate then swap ∈ with ∉, or vice versa
4. De Morgan's Law (Quantifier Negation)	~(x) Px	≡	(∃x) ~ Px	De Morgan's Law for a <u>single</u> quantifier
	~(∃x) Px	≡	(*x*) ~Px	
	~(x) (y) Pxy	≡	(∃x) (∃y) ~Pxy	De Morgan's Law for <u>nested</u> quantifiers
	~(x) (∃x) Pxy	≡	(∃x) (y) ~Pxy	
	~(∃x) (y) Pxy	≡	(x) (∃y) ~Pxy	
	~(∃x) (∃y) Pxy	≡	(x) (y) ~Pxy	
5. Nested / Multiple-Quantified Statements	(x) (y)	≡	(y) (x)	• for all objects x and y, …
	(∃x) (∃y)	≡	(∃y) (∃x)	• there are objects x and y such that …
	(x) (∃y) Pxy	≢	(∃x) (y) Pxy	False Counterexample for x, y ∈ ℤ: (x) (∃y) ($x + y = 0$) ≡ True (∃x) (y) ($x + y = 0$) ≡ False
	~((x) (∃y) Pxy)	≡	(∃x) (y) ~Pxy	Negation of multiple quantified statements
	~((∃x) (y) Pxy)	≡	(x) (∃y) ~Pxy	
6. Moving Quantifiers	(x) (Px ⊃ (∃y) Qxy)	≡	(x) (∃y) (Px ⊃ Qxy)	You can move a quantifier left if the variable is not used yet

Valid Quantifier Formulas

A		B
(x) (Px • Qx)	≡	(x) Px • (x) Qx
(∃x) (Px • Qx)	→	(∃x) Px • (∃x) Qx
(x) (Px ∨ Qx)	←	(x) Px ∨ (x) Qx
(∃x) (Px ∨ Qx)	≡	(∃x) Px) ∨ (∃x) Qx
(x) (Px ⊃ Qx)	←	(∃x) Px ⊃ (x) Qx
(∃x) (Px ⊃ Qx)	≡	(x) Px ⊃ (∃x) Qx
(x) ~Px	≡	~(∃x) Px
(∃x) ~Px	≡	~(x) Px
(x) (∃y) Txy	←	(∃y) (x) Txy
(y) (∃x) Txy	←	(∃x) (y) Txy
(x) (y) Txy	≡	(y) (x) Txy
(∃x) (∃y) Txy	≡	(∃y) (∃x) Txy
(x) (Px ∨ R)	≡	(x) Px ∨ R
(∃x) (Px • R)	≡	(∃x) Px • R
(x) (Px ⊃ R)	≡	(∃x) Px ⊃ R
(∃x) (Px ⊃ R)	→	(x) Px ⊃ R
(x) (R ⊃ Qx)	≡	R ⊃ (x) Qx
(∃x) (R ⊃ Qx)	→	R ⊃ (∃x) Qx
(x) R	←	R
(∃x) R	→	R

Note: The above formulas are valid in classical first-order logic, assuming that *x* does not occur free in *R*.

Invalid Quantifier Formulas

A		B	Counterexample
(∃x) (Px • Qx)	←	(∃x) Px • (∃x) Qx	D = {a, b}, M = {Pa, Qb}
(x) (Px ∨ Qx)	→	(x) Px ∨ (x) Qx	D = {a, b}, M = {Pa, Qb}
(x) (Px ⊃ Qx)	→	(∃x) Px ⊃ (x) Qx	D = {a, b}, M = {Pa, Qa}
(x) (∃y) Txy	→	(∃y) (x) Txy	D = {a, b}, M = {Tab, Tba}
(∃x) (Px ⊃ R)	←	(x) Px ⊃ R	D = ∅, M = {R}
(∃x) (R ⊃ Qx)	←	R ⊃ (∃x) Qx	D = ∅, M = ∅
(x) R	→	R	D = ∅, M = ∅
(∃x) R	←	R	D = ∅, M = {R}

Note: if empty domains are not allowed, then the last four implications above are in fact valid.

Sources

- Hurley, Patrick J. (2024). A Concise Introduction to Logic, 14th Edition, Cengage Learning, Inc.
- Wikipedia (2025).
  - https://en.wikipedia.org/wiki/List_of_logic_symbols
  - https://en.wikipedia.org/wiki/Truth_function#Table_of_binary_truth_functions

See Also

- Harold's Logic Cheat Sheet
- Harold's Logic (Philosophy) Cheat Sheet
- Harold's Sets Cheat Sheet
- Harold's Boolean Algebra Cheat Sheet
- Harold's Proofs Cheat Sheet