Harold's
Trigonometry and Hyperbolic Parent Functions
Cheat Sheet

AKA Library of Functions

8 January 2026

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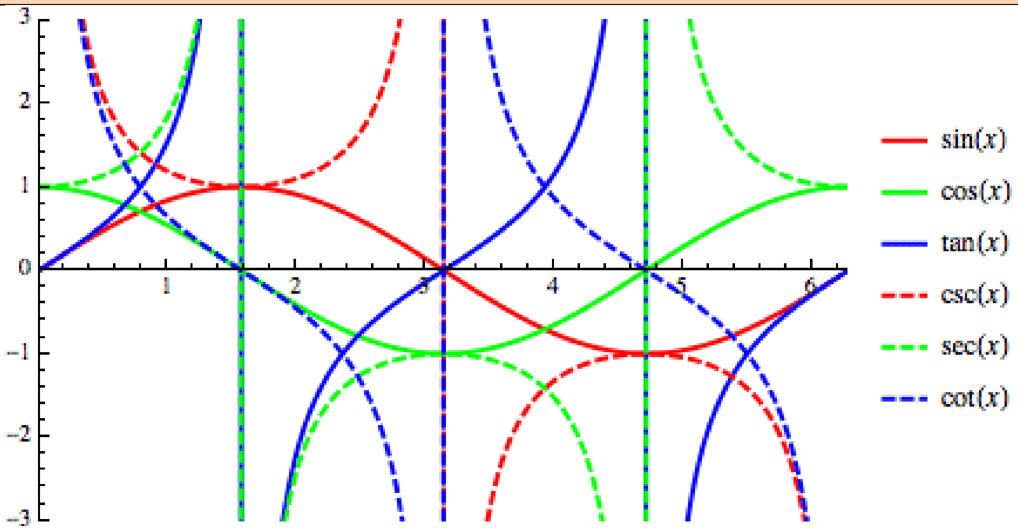
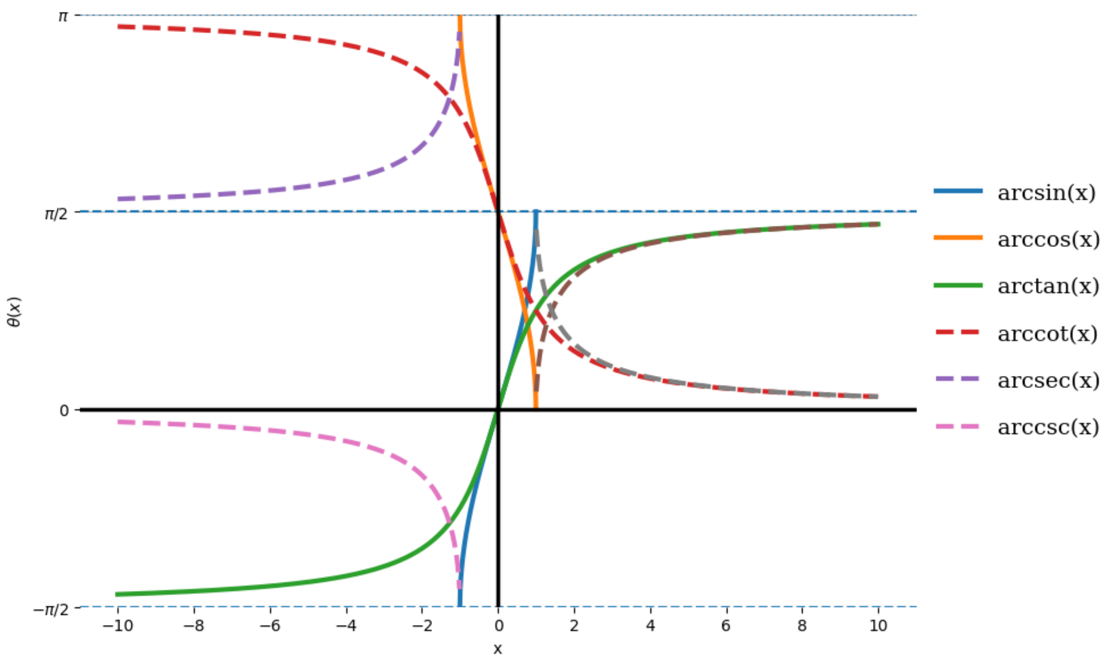
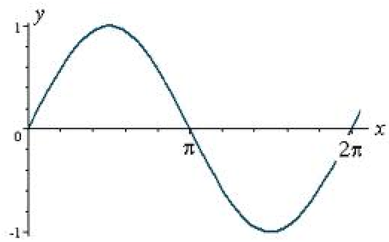
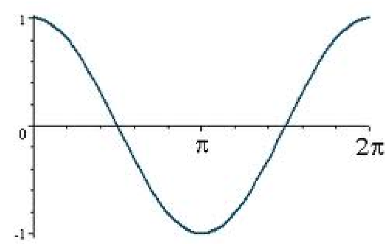
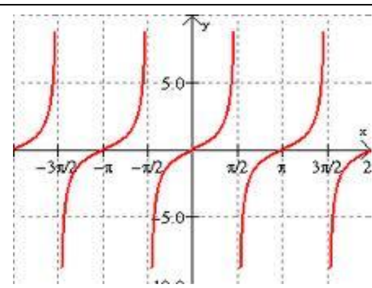
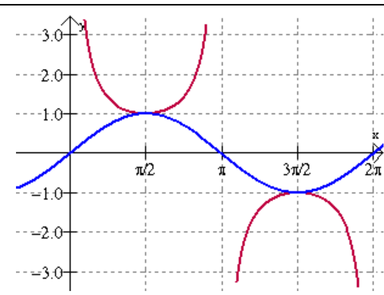
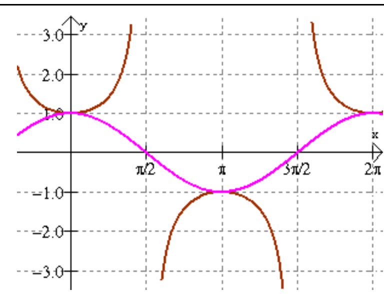
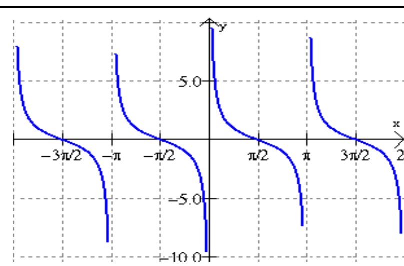
Trigonometric Functions



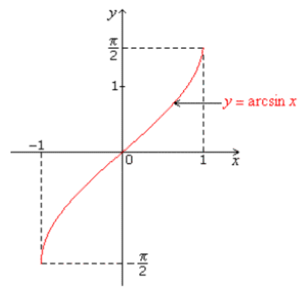
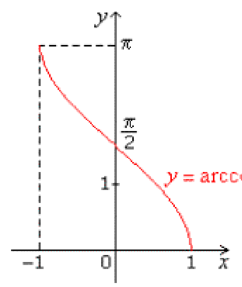
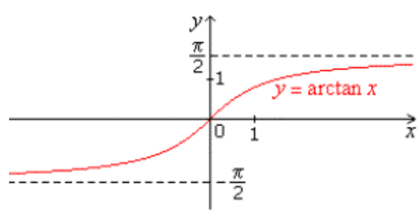
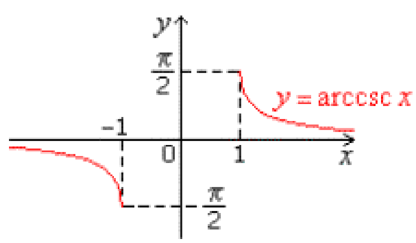
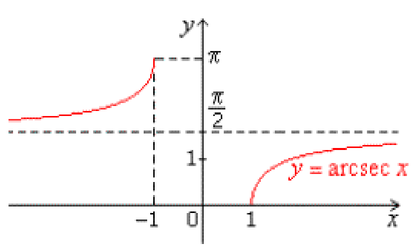
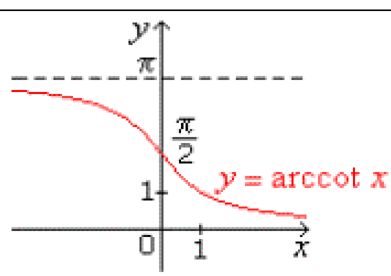
Inverse Trigonometric Functions



Function Name	Parent Function	Graph	Characteristics
Trigonometric			
Sine	$f(x) = sin\, x$		Domain: $(-\infty, \infty)$ with T = 2π/|b| Range: $[-1, 1]$ Inverse Function: $f^{-1}(x) = sin^{-1} x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\, sin\,\big(b(x-h)\big) + k$
Cosine	$f(x) = cos\, x$		Domain: $(-\infty, \infty)$ with T = 2π/|b| Range: $[-1, 1]$ Inverse Function: $f^{-1}(x) = cos^{-1} x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a\ cos\ \big(b(x-h)\big) + k$
Tangent	$f(x) = tan\, x$ $= \dfrac{sin\, x}{cos\, x}$		Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = tan^{-1} x$ Restrictions: Asymptotes at $x = \frac{\pi}{2} \pm n\pi$ Odd/Even: Odd General Form: $f(x) = a\, tan\ \big(b(x-h)\big) + k$
Cosecant	$f(x) = csc\, x$ $= \dfrac{1}{sin\, x}$		Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$ Range: (−∞, -1] ∪ [1, ∞) Inverse Function: $f^{-1}(x) = csc^{-1} x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a\ csc\ \big(b(x-h)\big) + k$
Secant	$f(x) = \sec x$ $= \dfrac{1}{cos\, x}$		Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$ Range: (−∞,−1] ∪ [1, ∞) Inverse Function: $f^{-1}(x) = sec^{-1} x$ Restrictions: Range is bounded Odd/Even: Even General Form: $f(x) = a\, sec\ \big(b(x-h)\big) + k$
Cotangent	$f(x) = cot\, x$ $= \dfrac{1}{tan\, x}$		Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = cot^{-1} x$ Restrictions: Asymptotes at x = $\pm n\pi$ Odd/Even: Odd General Form: $f(x) = a\, cot\ \big(b(x-h)\big) + k$

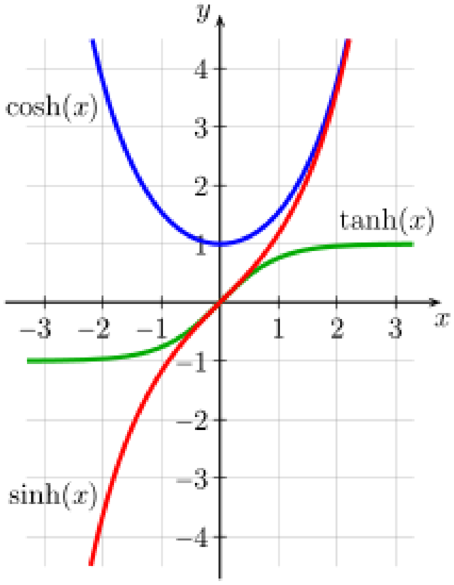
Function Name	Parent Function	Graph	Characteristics

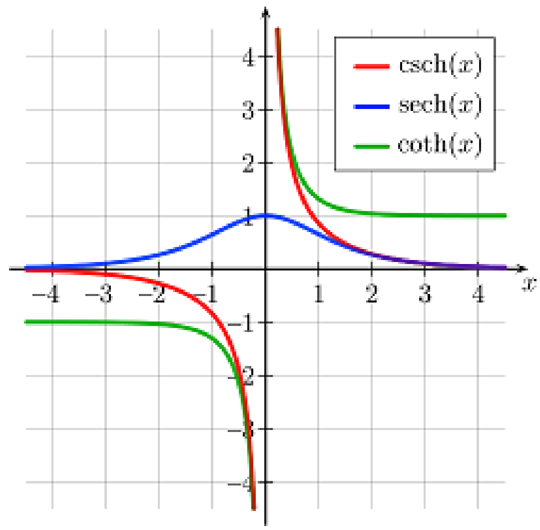
Inverse Trigonometric

Arcsine	$f(x) = sin^{-1}x$		Domain: $[-1, 1]$ Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = sin\,x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a\ sin^{-1}\left(b(x-h)\right) + k$
Arccosine	$f(x) = cos^{-1}x$		Domain: $[-1, 1]$ Range: $[0,\ \pi]$ or Quadrants I & II Inverse Function: $f^{-1}(x) = cos\,x$ Restrictions: Range & Domain are bounded Odd/Even: None General Form: $f(x) = a\ cos^{-1}\left(b(x-h)\right) + k$
Arctangent	$f(x) = tan^{-1}x$		Domain: $(-\infty, \infty)$ Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = tan\,x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a\,tan^{-1}\left(b(x-h)\right) + k$
Arccosecant	$f(x) = csc^{-1}x$		Domain: $(-\infty,-1] \cup [1, \infty)$ Range: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ or Quadrants I & IV Inverse Function: $f^{-1}(x) = csc\,x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a\ csc^{-1}\left(b(x-h)\right) + k$
Arcsecant	$f(x) = sec^{-1}x$		Domain: $(-\infty,-1] \cup [1, \infty)$ Range: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ or Quadrants I & II Inverse Function: $f^{-1}(x) = sec\,x$ Restrictions: Range & Domain are bounded Odd/Even: Neither General Form: $f(x) = a\,sec^{-1}\left(b(x-h)\right) + k$
Arccotangent	$f(x) = cot^{-1}x$		Domain: $(-\infty, \infty)$ Range: $(0, \pi)$ or Quadrants I & II Inverse Function: $f^{-1}(x) = cot\,x$ Restrictions: Range is bounded Odd/Even: Neither General Form: $f(x) = a\,cot^{-1}\left(b(x-h)\right) + k$

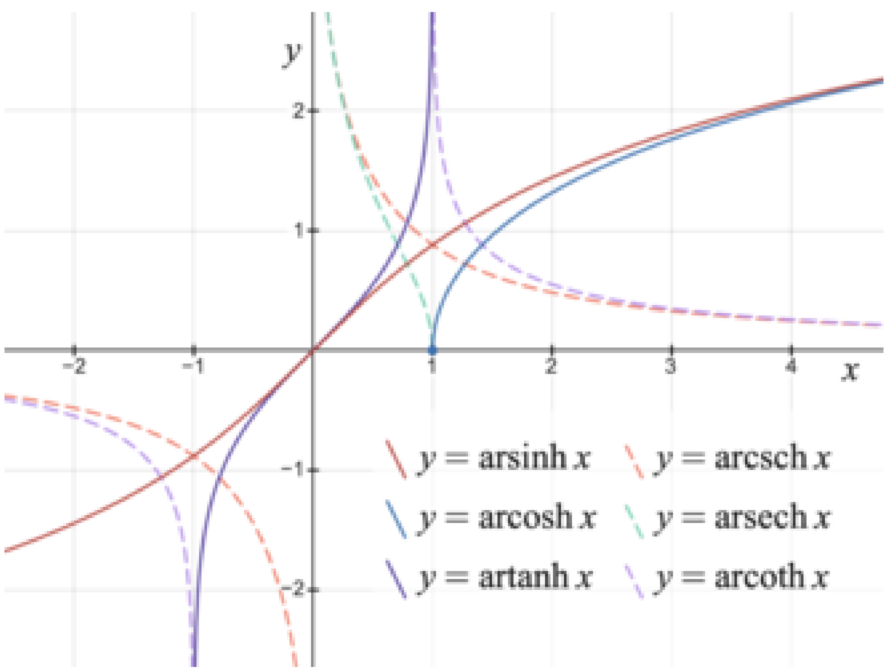
Function Name	Parent Function	Graph	Characteristics

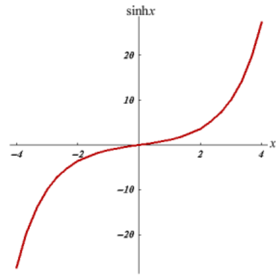
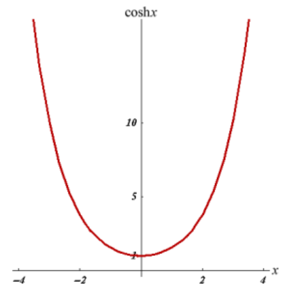
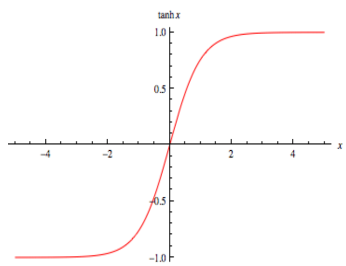
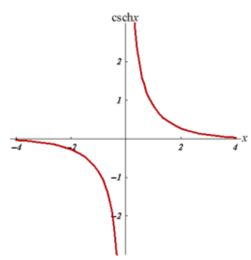
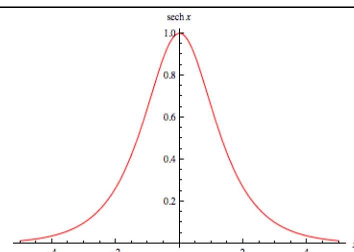
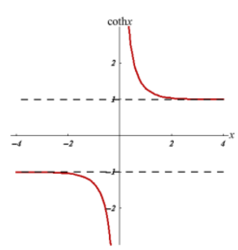
Hyperbolic Graphs

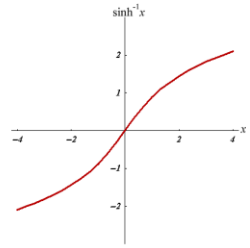
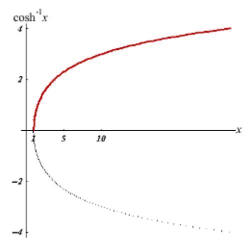
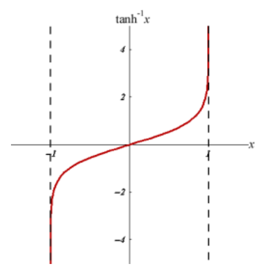
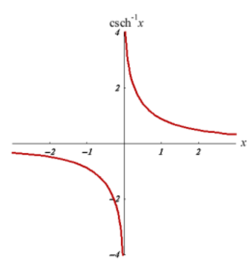
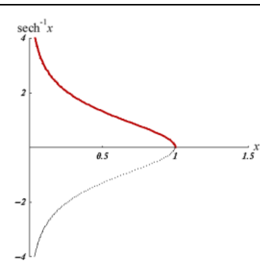
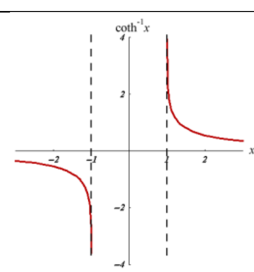




Inverse Hyperbolic Graphs



Function Name	Parent Function	Graph	Characteristics
Hyperbolic			
Hyperbolic Sine	$f(x) = \sinh x$ $= \dfrac{e^x - e^{-x}}{2}$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = sinh^{-1}\,x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\ sinh\left(b(x-h)\right) + k$
Hyperbolic Cosine	$f(x) = cosh\,x$ $= \dfrac{e^x + e^{-x}}{2}$		Domain: $(-\infty, \infty)$ Range: $[1, \infty)$ Inverse Function: $f^{-1}(x) = cosh^{-1}\,x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a\ \ cosh\ \left(b(x-h)\right) + k$
Hyperbolic Tangent	$f(x) = tanh\,x$ $= \dfrac{e^{2x} - 1}{e^{2x} + 1}$		Domain: $(-\infty, \infty)$ Range: $(-1, 1)$ Inverse Function: $f^{-1}(x) = tanh^{-1}\,x$ Restrictions: Asymptotes at $y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\,tanh\ \left(b(x-h)\right) + k$
Hyperbolic Cosecant	$f(x) = csch\,x$ $= \dfrac{1}{sinh\,x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0] \cup [0, \infty)$ Inverse Function: $f^{-1}(x) = csch^{-1}\,x$ Restrictions: Asymptotes at $x = 0, y = 0$ Odd/Even: Odd General Form: $f(x) = a\ \ csch\ \left(b(x-h)\right) + k$
Hyperbolic Secant	$f(x) = \mathrm{sech}\,x$ $= \dfrac{1}{cosh\,x}$		Domain: $(-\infty, \infty)$ Range: $(0, 1]$ Inverse Function: $f^{-1}(x) = sech^{-1}\,x$ Restrictions: Asymptote at $y = 0$ Odd/Even: Even General Form: $f(x) = a\,sech\left(b(x-h)\right) + k$
Hyperbolic Cotangent	$f(x) = coth\,x$ $= \dfrac{1}{tanh\,x}$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 1) \cup (1, \infty)$ Inverse Function: $f^{-1}(x) = coth^{-1}\,x$ Restrictions: Asymptotes at $x = 0, y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\,coth\left(b(x-h)\right) + k$

Function Name	Parent Function	Graph	Characteristics
Inverse Hyperbolic			
Hyperbolic Arcsine	$f(x) = sinh^{-1} x$ $= ln(x + \sqrt{x^2 + 1})$		Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = sinh\, x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\, sinh^{-1}\big(b(x-h)\big) + k$
Hyperbolic Arccosine	$f(x) = cosh^{-1} x$ $= ln(x + \sqrt{x^2 - 1})$		Domain: $[1, \infty)$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = cosh\, x$ Restrictions: $y \geq 0$ Odd/Even: Neither General Form: $f(x) = a\ cosh^{-1}\ \big(b(x-h)\big) + k$
Hyperbolic Arctangent	$f(x) = tanh^{-1} x$ $= \frac{1}{2} ln\left(\frac{1+x}{1-x}\right)$		Domain: $(-1, 1)$ Range: $(-\infty, \infty)$ Inverse Function: $f^{-1}(x) = tanh\, x$ Restrictions: Asymptotes at $x = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\, tanh^{-1}\ \big(b(x-h)\big) + k$
Hyperbolic Arccosecant	$f(x) = csch^{-1} x$ $= ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)$		Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0] \cup [0, \infty)$ Inverse Function: $f^{-1}(x) = csch\, x$ Restrictions: Asymptotes at $x = 0, y = 0$ Odd/Even: Odd General Form: $f(x) = a\ csch^{-1}\big(b(x-h)\big) + k$
Hyperbolic Arcsecant	$f(x) = sech^{-1} x$ $= ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$		Domain: $(0, 1]$ Range: $[0, \infty)$ Inverse Function: $f^{-1}(x) = sech\, x$ Restrictions: Odd/Even: Neither General Form: $f(x) = a\, sech^{-1}\big(b(x-h)\big) + k$
Hyperbolic Arccotangent	$f(x) = coth^{-1} x$ $= \frac{1}{2} ln\left(\frac{x+1}{x-1}\right)$		Domain: $[-\infty, -1) \cup (1, \infty]$ Range: $(-\infty, 0) \cup (0, \infty)$ Inverse Function: $f^{-1}(x) = coth\, x$ Restrictions: Asymptotes at $x = 0, y = \pm 1$ Odd/Even: Odd General Form: $f(x) = a\, coth^{-1}\ \big(b(x-h)\big) + k$

Graphing Tips

$$f(x) = a < trig > (b(x - h)) + k$$

The Six Trig "Levers"	*y = a sin (b (x - h)) + k*	Graphing Tips	Notes
1) Move up/down ↕	k (Vertical translation)	$k = \frac{(\max + \min)}{2}$	If $k = f(x)$ then x-axis is replaced by $f(x)$-axis
2) Move left/right ↔	h (Phase shift)	"+" shifts right	$sin\ (x) = cos\ (x - \pi/2)$
3) Stretch up/down ↕	a (Amplitude)	$a = \frac{(\max - \min)}{2}$	a is NOT peak-to-peak on y-axis
4) Stretch left/right ↔	b (Frequency • 2π)	$T = \frac{2\pi}{\lvert b \rvert} = \frac{1}{f}$	T = peak-to-peak on θ-axis $T = \frac{\pi}{\lvert b \rvert}$ for $tan\ (bx)$
5) Flip about the y-axis ↺	b → −b	$f(x) \to f(-x)$	Even Function: $cos\ (x) = cos\ (-x)$
6) Flip about the x-axis ↻	a → −a	$f(x) \to -f(-x)$	Odd Function: $sin\ (x) = -sin\ (-x)$

See Also:

- Harold's Algebraic, Transcendental, and Conic Parent Functions Cheat Sheet
- Harold's Trigonometry & Hyperbolic Parent Functions Cheat Sheet
- Harold's Polar Parent Functions Cheat Sheet