# Harold's Partial Fraction Decomposition (Precalculus) Cheat Sheet

20 January 2026

| Partial Fractions | (http://en.wikipedia.org/wiki/Partial_fraction_decomposition) |
|---|---|
| **Condition** | $f(x) = \frac{P(x)}{Q(x)} = \frac{ax^n + \cdots + b}{cx^m + \cdots + d}$ where $P(x)$ *and* $Q(x)$ are polynomials |
| **Preparation** | Case 1: $n \geq m$, Perform long division first<br>Case 2: $n < m$, Proceed to the cases below |
| **Case I: Simple linear ($1^{st}$ degree)** | $\frac{A}{(ax+b)}$ $e.g.,$ $\frac{A}{x}$, $\frac{A}{x+1}$ |
| **Case II: Multiple degree linear ($1^{st}$ degree)** | $\frac{A}{(ax+b)} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3}$ |
| **Case III: Simple quadratic ($2^{nd}$ degree)** | $\frac{Ax+B}{(ax^2+bx+c)}$ $e.g.,$ $\frac{3x}{(x^2+9)}$ |
| **Case IV: Multiple degree quadratic ($2^{nd}$ degree)** | $\frac{Ax+B}{(ax^2+bx+c)} + \frac{Cx+D}{(ax^2+bx+c)^2} + \frac{Ex+F}{(ax^2+bx+c)^3}$ |

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| **Example Expansion** | $\frac{P(x)}{(ax+b)(cx+d)^2(ex^2+fx+g)}$<br>$= \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2} + \frac{Dx+E}{(ex^2+fx+g)}$ |

| Steps to Solve | Precalculus Example |
|---|---|
| **1. Write down problem** | $\frac{5x+1}{2x^2-x-1}$ |
| **2. Check if long division is needed** | Not needed since degree of numerator (top) is less than degree of denominator (bottom) |
| **3. Factor the denominator** | $\frac{5x+1}{(2x+1)(x-1)}$ |
| **4. Expand with A, B, Cs** | $\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{(2x+1)} + \frac{B}{(x-1)}$ |
| **5. Find a common denominator** | $= \frac{A(x-1)}{(2x+1)(x-1)} + \frac{B(2x+1)}{(2x+1)(x-1)}$ |
| **6. Focus on numerator** | $5x+1 = A(x-1) + B(2x+1)$ |
| **7. FOIL if necessary** | $(x+1)(x-2) = x^2 - x - 2$ |

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| **8. Expand/distribute the A, B, Cs** | $5x + 1 = Ax - A + 2Bx + B$ |
| **9. Regroup by powers of x.** $(x^2, x, c)$ | $5x + 1 = Ax + 2Bx - A + B$ |
| **10. Factor by powers of x.** $()x^2 + ()x + (c)$ | $(5)x + (1) = (A + 2B)x + (-A + B)$ |
| **11. Introduce ghost factors if needed** (0, 1) | $5x + 1 = (0)x^2 + (5)x + (1)$ |
| **12. Match left and right coefficients for a system of equations** | $A + 2B = 5$<br>$-A + B = 1$ |
| **13. Solve system of equations** | Pick simplest method below |
| **a. Substitution method** | $B = A + 1$<br>$A + 2(A + 1) = 5$<br>$A + 2A + 2 = 5$<br>$3A = 3$<br>$A = 1$<br>$B = A + 1 = 1 + 1 = 2$<br>$\boldsymbol{A = 1}$<br>$\boldsymbol{B = 2}$ |
| **b. Row elimination method** | $A + 2B = 5$<br>$+ [-A + B = 1]$<br>----------<br>$3B = 6$<br>$\boldsymbol{B = 2}$<br><br>$A + 2B = 5$<br>$-2\ [-A + B = 1]$<br>----------<br>$3A = 3$<br>$\boldsymbol{A = 1}$ |
| **c. Augmented matrix method** | $\left[\begin{array}{cc\|c} A & B & k \\ A & B & k \end{array}\right] = \left[\begin{array}{cc\|c} 1 & 2 & 5 \\ -1 & 1 & 1 \end{array}\right]$<br><br>Use TI-84 $rref()$ function<br><br>$= \left[\begin{array}{cc\|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$<br><br>$\boldsymbol{A = 1}$<br>$\boldsymbol{B = 2}$ |
| **14. Reassemble the newly expanded function with values for A, B, C** | $\frac{5x + 1}{(2x + 1)(x - 1)} = \frac{1}{(2x + 1)} + \frac{2}{(x - 1)}$ |
| **15. Verify function for accuracy** | *Verify the two equations are the same by plugging in any value for x and see if f(x) is the same for both.* |

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