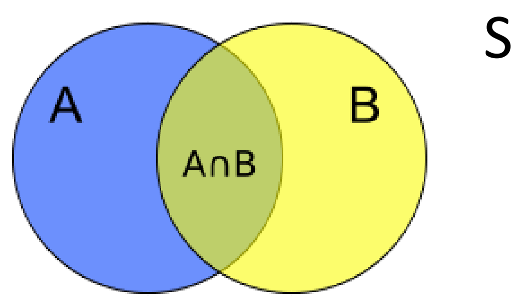
Harold's Probability
Cheat Sheet
23 October 2025

Probability

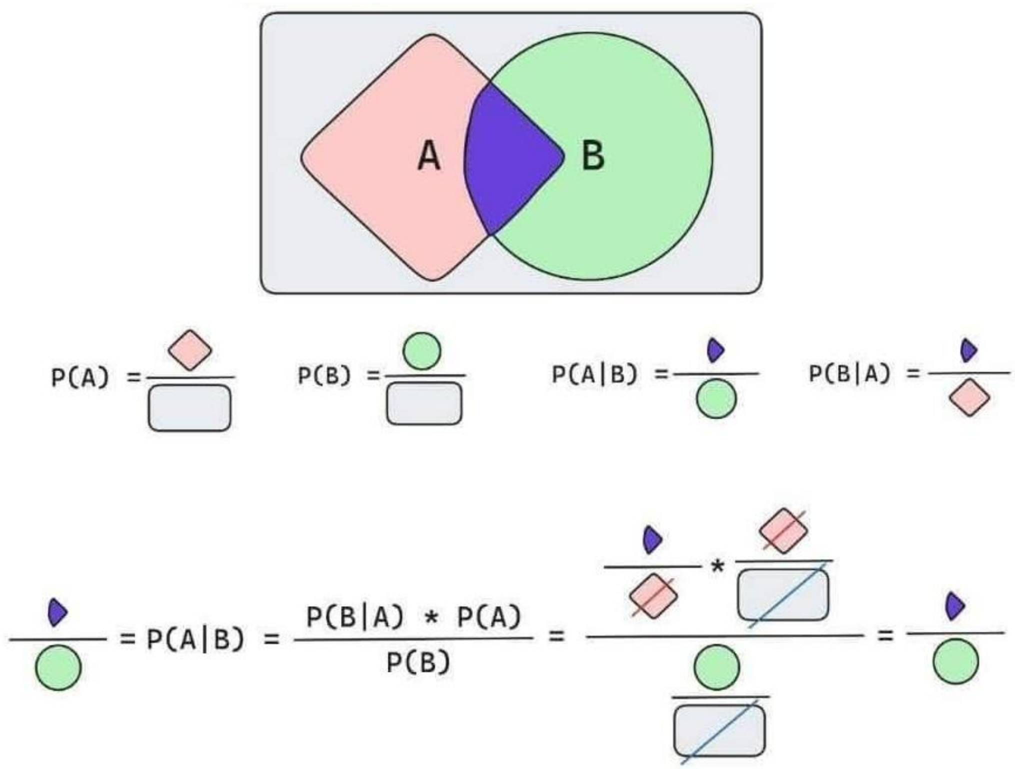
Rule	Formula	Definition
Notation	∩ = "and", Intersection, "∧" ∪ = "or", Union, "∨" $\overline{\square}$ = "not", negation, "¬"	"and" implies multiplication. "or" implies addition. "not" implies negation.
Independent	If $P(A\|B) = P(A)$	The occurrence of one event does not affect the probability of the other, or vice versa.
Dependent	If $P(A \cap B) \neq \emptyset$	The occurrence of one event affects the probability of the other event.
Disjoint ("mutually exclusive")	If $P(A \cap B) = \emptyset$ Then $\boldsymbol{P(A \cup B) = P(A) + P(B)}$	The events can never occur together.
Probability ("likelihood")	$\boldsymbol{0 \leq P(E) \leq 1}$ $P(E) = \frac{\#\ Events\ (E)}{Sample\ Space\ (S)} = \frac{\#\ of\ Favorable\ Outcomes}{Total\ \#\ of\ Possible\ Outcomes}$	
Addition Rule ("or")	$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$	

Multiplication Rule
("and")
	if independent or disjoint:
	$\boldsymbol{P(A \cap B) = P(A)\ P(B)}$
	$\boldsymbol{P(A \cap B \cap C) = P(A)\ P(B)\ P(C)}$
	
	if dependent:
	$\boldsymbol{P(A \cap B) = P(A)\ P(B\|A)}$
	$P(A \cap B) = P(B)\ P(A\|B)$
	$P(A \cap B) = P(A) - P(A \cap \overline{B})$	

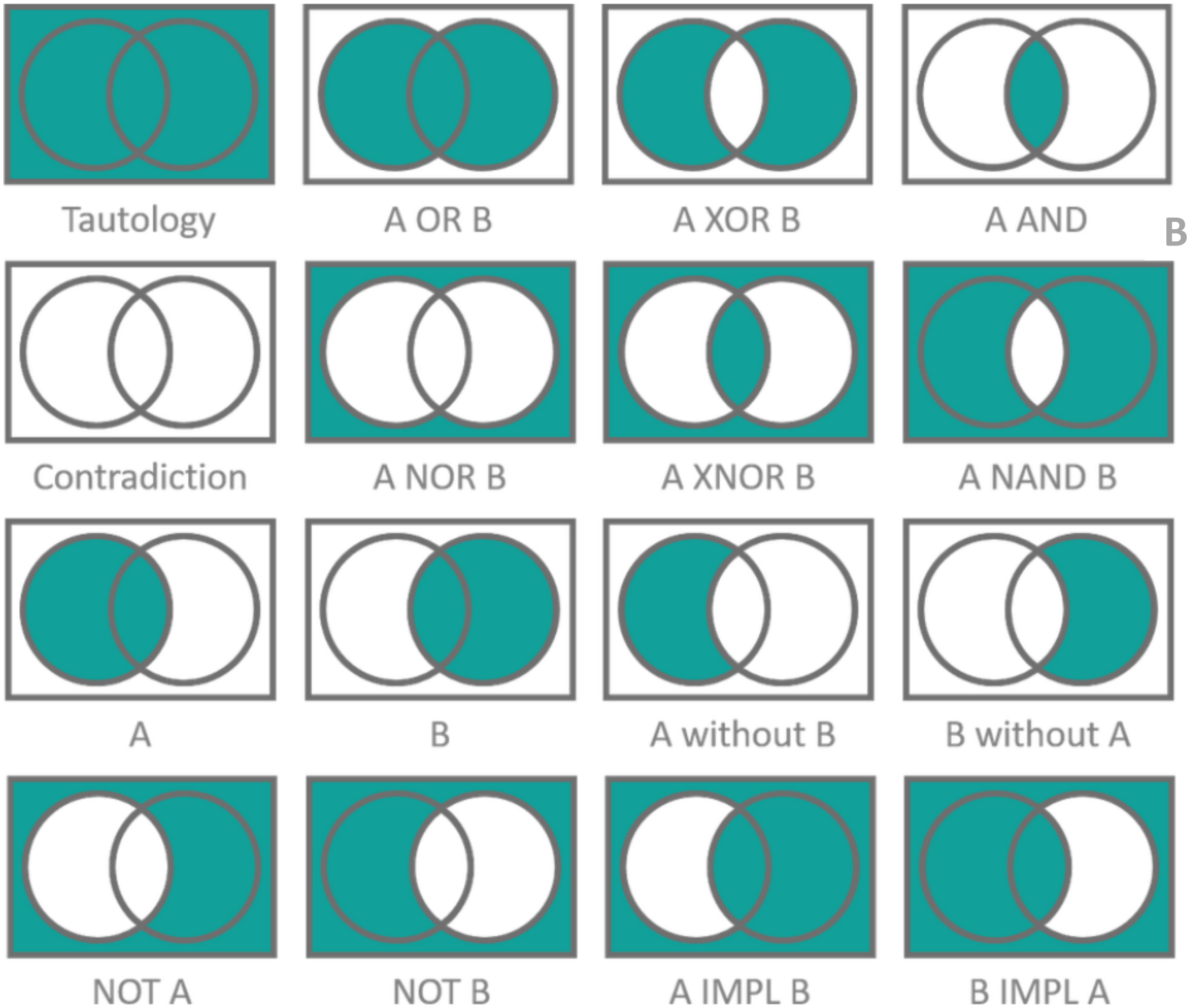
Complement Rule / Subtraction Rule ("not")	$\boldsymbol{P(S)} = P(A \cup \overline{A}) =$ $P(A) + P(\overline{A}) = \boldsymbol{1}$ $P(A) = 1 - P(\overline{A})$ $\boldsymbol{P(\overline{A}) = 1 - P(A)}$ $P(A\|B) + P(\overline{A}\|B) = 1$	The complement of event A (denoted $\overline{A}\ or\ A^c$) means "**not A**"; it consists of all simple outcomes that are not in A.
Conditional Probability ("given that")	$\boldsymbol{P(A\|B) = \frac{P(A \cap B)}{P(B)}}$ if independent or disjoint: $P(A\|B) = P(A)$ $P(B\|A) = P(B)$	Means the probability of event A given that event B occurred. Is a rephrasing of the Multiplication Rule. P(A|B) is the proportion of elements in B that are ALSO in A.

Total Probability Rule	$P(A) = P(A \cap B_1) + \cdots + P(A \cap B_n)$ $= P(B_1)\, P(A\|B_1) + \cdots + P(B_n)\, P(A\|B_n)$ $P(A) = P(A \cap B) + P(A \cap \bar{B})$ $= P(B)\, P(A\|B) + P(\bar{B})\, P(A\|\bar{B})$	To find the probability of event A, partition the sample space into several disjoint events. A must occur along with one and only one of the disjoint events.
Bayes' Theorem	$\boldsymbol{P(A\|B)} = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\, P(B\|A)}{P(B)}$ $= \frac{\boldsymbol{P(A)\, P(B\|A)}}{\boldsymbol{P(A)\, P(B\|A) + P(\bar{A})\, P(B\|\bar{A})}}$	Allows P(A|B) to be calculated from P(B|A). Meaning it allows us to reverse the order of a conditional probability statement, and is the only generally valid method!
De Morgan's Law	$\overline{P(A \cup B)} \equiv \overline{P(A)} \cap \overline{P(B)}$ $\overline{P(A \cap B)} \equiv \overline{P(A)} \cup \overline{P(B)}$	Uses negation to convert an "or" to an "and". Uses negation to convert an "and" to an "or".

Visual Proof of Bayes' Theorem



Venn Diagrams

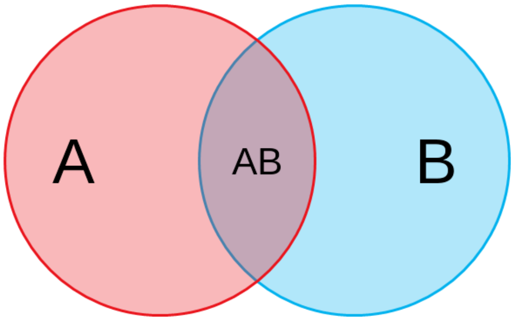


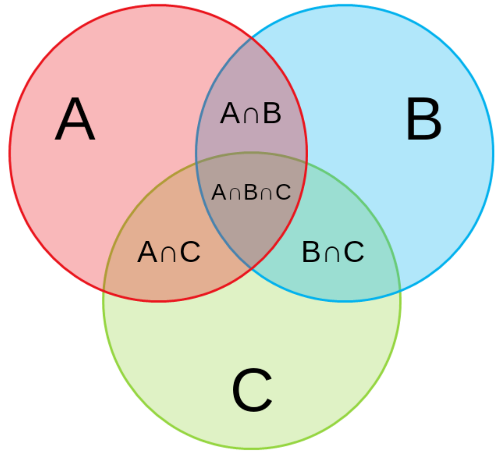
Sources

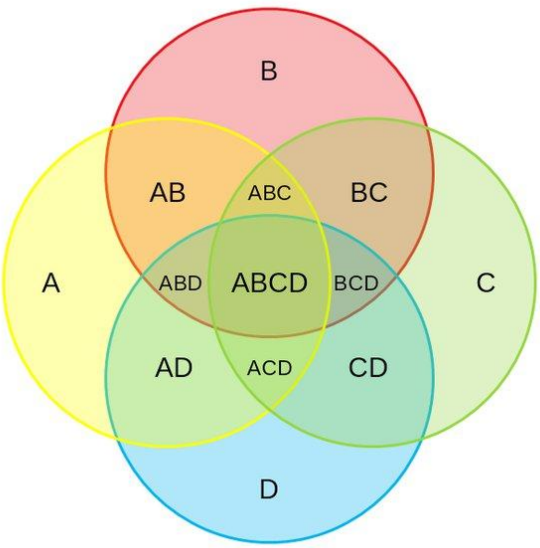
- SNHU MAT 229 - Mathematical Proof and Problem Solving, How To Prove It - A Structured Approach, 3rd Edition - Daniel J. Vellman, Cambridge University Press, 2019.

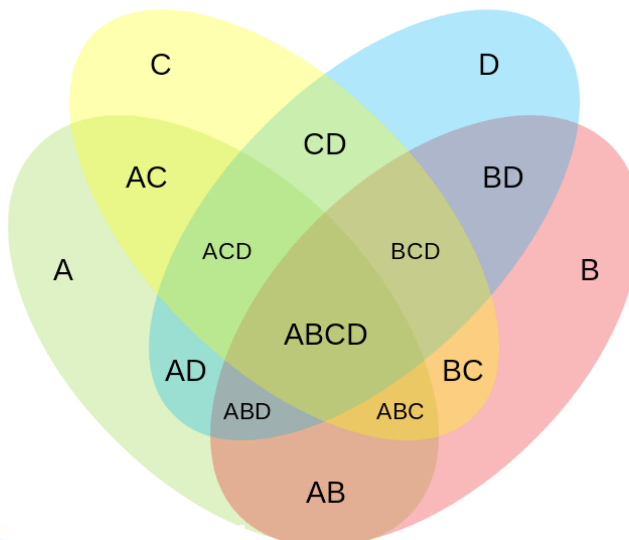
- SNHU MAT 230 - Discrete Mathematics, zyBooks.

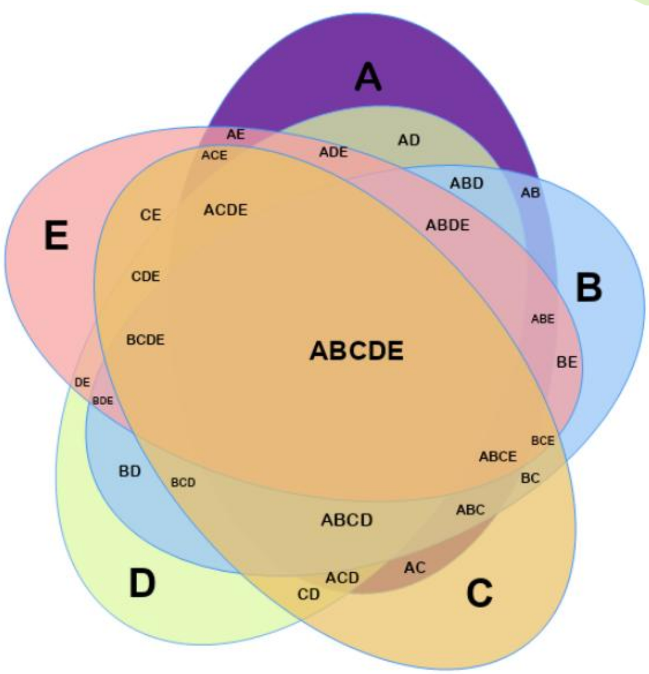
- Learn more about probability distributions here: http://blog.cloudera.com/blog/2015/12/common-probability-distributions-the-data-scientists-crib-sheet/


A
AB
B


A
A∩B
B
A∩B∩C
A∩C
B∩C
C


B
AB
ABC
BC
A
ABD
ABCD
BCD
C
AD
ACD
CD
D


C
D
CD
AC
BD
ACD
BCD
A
B
ABCD
AD
BC
ABD
ABC
AB


A
AE
ACE
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CDE
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ABE
BCDE
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DE
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BCE
ABCE
BD
BCD
BC
ABC
ABCD
AC
ACD
CD
D
C