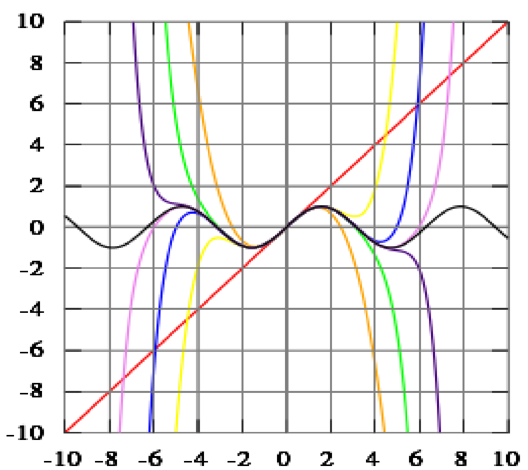
# Harold's Taylor Series Cheat Sheet

1 April 2026

| Power Series | |
|---|---|
| **Power Series About Zero**<br>Geometric Series if $a_n = a$ | $\sum_{n=0}^{+\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$ |
| **Power Series** | $\sum_{n=0}^{+\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \cdots$ |

| Approximation Polynomial | |
|---|---|
|  | $\boldsymbol{f(x) = P_n(x) + R_n(x)}$<br><br>$P_n(x) = n^{th}$ degree polynomial approximation<br>$R_n(x) = \pm\, Error$<br><br>NOTE: $P_n(x)$ is easy to integrate and differentiate |

| Maclaurin Series | |
|---|---|
| **Maclaurin Series**<br>Taylor Series centered about $x = 0$ | $\boldsymbol{f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n}$ |
| **Maclaurin Series Remainder** | $R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!} x^{n+1}$<br>where $x \le x^* \le max$ and $\lim_{x \to +\infty} R_n(x) = 0$ |

| Taylor Series | |
|---|---|
| **Taylor Series**<br>Maclaurin Series if $c = 0$ | $f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$ |
| **Taylor Series Remainder** | $R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!} (x-c)^{n+1}$<br>where $x \le x^* \le c$ and $\lim_{x \to +\infty} R_n(x) = 0$ |

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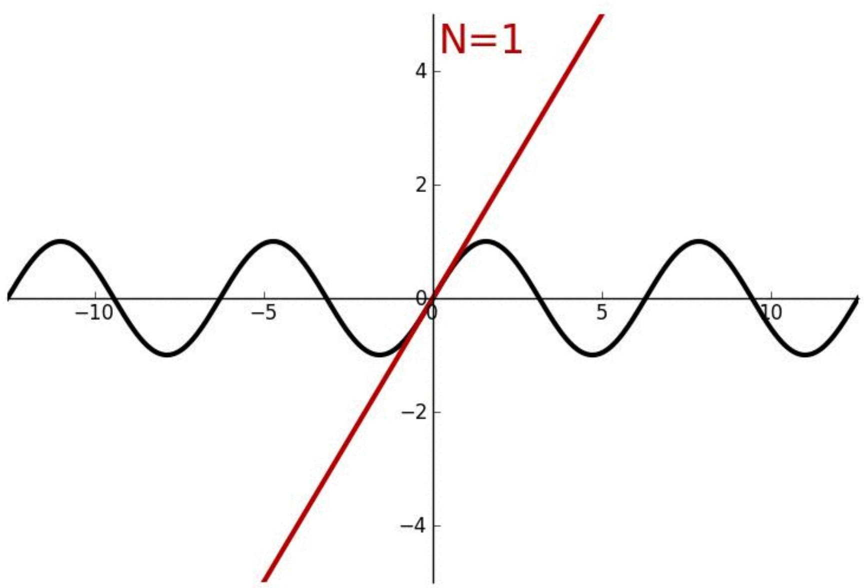
| Key Maclaurin Series | Expanded Form |
|---|---|
| $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all $x$ | $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \cdots$ |
| $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$ for $\lvert x\rvert < 1$ | $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \cdots$ |
| $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $\lvert x\rvert < 1$ | $1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + \cdots$ |
| $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for all $x$ | $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \frac{x^{15}}{15!} + \cdots$ |
| $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ for all $x$ | $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \frac{x^{14}}{14!} + \cdots$ |
| $\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} x^{2n+1}$<br>for $\lvert x\rvert < 1$ | $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots \; for -1 < x < 1$<br>$\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \frac{1}{9x^9} + \cdots \; for\ x \geq 1$<br>$-\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \frac{1}{9x^9} + \cdots \; for\ x < 1$ |
| $\sinh(x) = \frac{e^x - e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ for all $x$ | $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \frac{x^{11}}{11!} + \frac{x^{13}}{13!} + \frac{x^{15}}{15!} + \cdots$ |
| $\cosh(x) = \frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ for all $x$ | $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \frac{x^{10}}{10!} + \frac{x^{12}}{12!} + \frac{x^{14}}{14!} + \cdots$ |




See Harold's Infinite Series Cheat Sheet

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