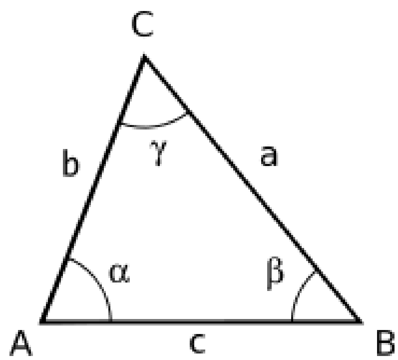
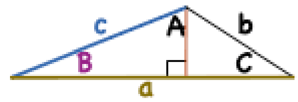
Harold's Triangles Cheat Sheet

21 January 2026

Trig Laws and Formulas

Law	Equation
Reference Triangle	
Law of Sines	$\boldsymbol{\frac{sin(A)}{a} = \frac{sin(B)}{b} = \frac{sin(C)}{c}}$ $\frac{a}{sin(A)} = \frac{b}{sin(B)} = \frac{c}{sin(C)}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cdot cos(A)$ $b^2 = a^2 + c^2 - 2ac \cdot cos(B)$ $\boldsymbol{c^2 = a^2 + b^2 - 2ab \cdot cos(C)}$
Law of Tangents	$\frac{a-b}{a+b} = \frac{tan\left[\frac{(A-B)}{2}\right]}{tan\left[\frac{(A+B)}{2}\right]}$ $\frac{b-c}{b+c} = \frac{tan\left[\frac{(B-C)}{2}\right]}{tan\left[\frac{(B+C)}{2}\right]}$ $\frac{a-c}{a+c} = \frac{tan\left[\frac{(A-C)}{2}\right]}{tan\left[\frac{(A+C)}{2}\right]}$ $tan(A) \cdot tan(B) \cdot tan(C) = tan(A) + tan(B) + tan(C)$
Law of Cotangents	$\frac{cot\left(\frac{A}{2}\right)}{s-a} = \frac{cot\left(\frac{B}{2}\right)}{s-b} = \frac{cot\left(\frac{C}{2}\right)}{s-c}$ $cot(A) \cdot cot(B) \cdot cot(C) = cot(A) + cot(B) + cot(C)$

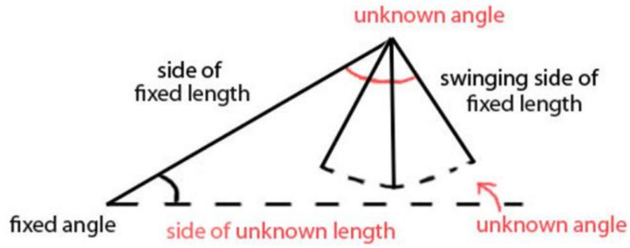
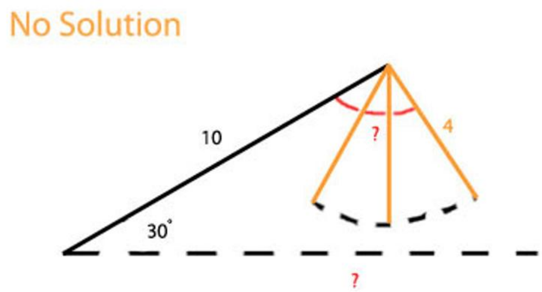
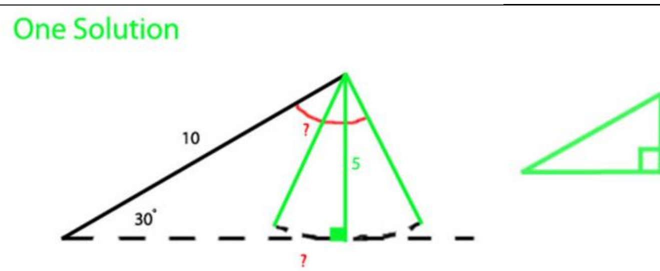
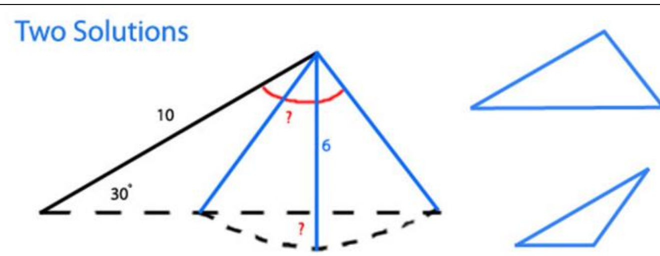
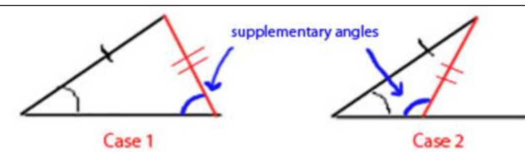
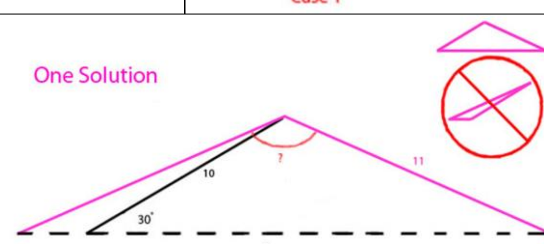
Pythagorean Theorem	If a right triangle, then $$c^2 = a^2 + b^2$$ **Special Case**: Same as Law of Cosines with angle C = 90°. $$c^2 = a^2 + b^2 - 2ab \cdot cos(90°)$$ $$c^2 = a^2 + b^2 - 2ab \cdot 0 = a^2 + b^2$$	
Sum of Angles	$A° + B° + C° = 180°$ $C° = 180° - (A° + B°)$	$A + B + C = \pi\ radians$ $C = \pi - (A + B)$
Area Formula	$A = \frac{1}{2}bh = \frac{1}{2}ab \cdot sin(C)$	
Semi-Perimeter	$s = \frac{a + b + c}{2}$	
Heron's Formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$	
Mollweide's Formula	$\frac{a + b}{c} = \frac{cos\left[\frac{(A-B)}{2}\right]}{sin\left(\frac{C}{2}\right)}$	

Solving for Angles, Sides, and Area

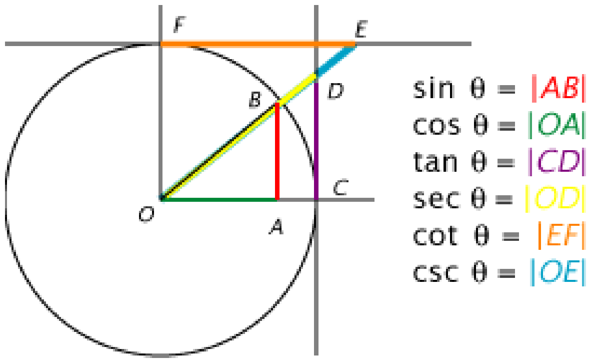
Order of Use	Comments
1. Sum of Angles	Easiest formula.
2. Pythagorean Theorem	Use if one of the angles is a right angle (90°).
3. Law of Sines	Least complex. Use before Law of Cosines, if possible.
4. Law of Cosines	More complex. Use only once, then use Law of Sines.
5. Heron's Formula	Use for area if all three sides are known.
6. Law of Tangents	Very complex and seldom used.
7. Law of Cotangents	Most complex and seldom used.

Given	Find		
	Angle	Side	Area
SSS	Law of Cosines	Given	Heron's Formula
SAS	NA	Law of Cosines	$A = \frac{1}{2}ab \cdot sin(C)$
SSA	Law of Sines	NA	$A = \frac{1}{2}bh$
ASS			
SAA	Sum of Angles	Law of Sines	
ASA			
AAS			
AAA	Given	Unsolvable. Not unique. One side needed.	

Ambiguous Cases for SSA

Scenario	# of Solutions	Illustrations
SSA	**0 – 2 Solutions**	 $h = a\sin\theta$
$\boldsymbol{b < h}$	**No Solution**	
$\boldsymbol{b = h}$	**One Solution**	
$\mathbf{b > h}$	**Two Solutions**	 $\frac{b}{sin\,(B)} = \frac{b}{sin\,(180° - B)}$ 
a ≥ b	**One Oblique Solution**	 When *a* = *b*, equilateral / isososocles When *a* > *b*, obtuse

Interesting Trig Lengths on a Unit Circle

<table>
<tr><th colspan="2">Trig Graphically</th></tr>
<tr><td>


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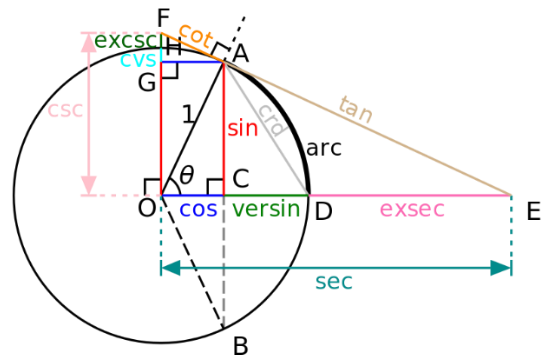

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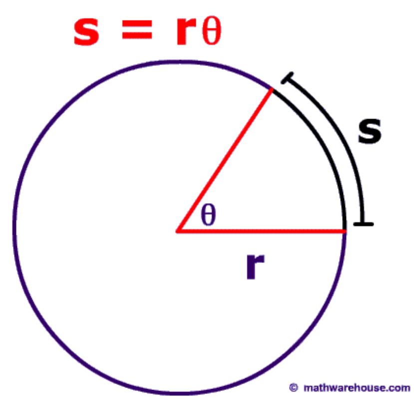
<tr><th>Radians and Arc Length</th><th></th></tr>
<tr><td>
Radian = arc length (s) of a unit circle
$s = r\,\theta$
$C = \pi D = \pi(2r) = 2\pi r$

Proof:
If r = 1 (unit circle)
then $s = (1)\theta = \theta$ and $C = 2\pi\,(1) = 2\pi$
Therefore, $360° = 2\pi$ radians

To convert degrees to radians:
$n° \times \left(\frac{\pi \text{ rad}}{180°}\right) = m$ radians

To convert radians to degrees:
$m \text{ rad} \times \left(\frac{180°}{\pi \text{ rad}}\right) = n°$
</td><td>


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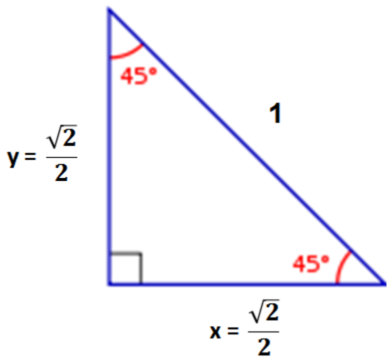
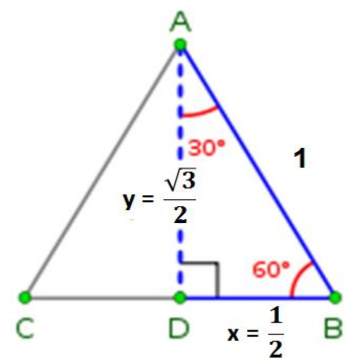
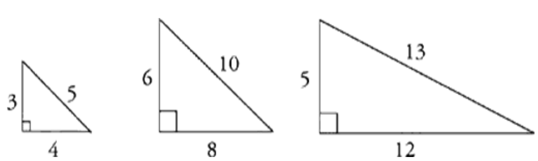
Sources

- SAS Triangle images. https://mathimages.swarthmore.edu/index.php/Ambiguous_Case

Fixed Angles Triangles

45-45-90 Triangle	30-60-90 Triangle
	
Proof: $a^2 + b^2 = c^2$ $x = y$ $x^2 + x^2 = 1^2$ $2x^2 = 1$ $x^2 = ½$ $\sqrt{x^2} = \sqrt{\frac{1}{2}}$ $x = \frac{\sqrt{2}}{2}$	**Proof:** $a^2 + b^2 = c^2$ $x = ½c$ $y^2 + (½)^2 = 1^2$ $y^2 + ¼ = 1$ $y^2 = ¾$ $\sqrt{y^2} = \sqrt{\frac{3}{4}}$ $y = \frac{\sqrt{3}}{2}$

Fixed Sided Triangles

Pythagorean Triples			
A Pythagorean triple is a right triangle with only integer sides. The examples on the right are expressed in the lowest form. You can scale any of these with an integer (e.g., 2,3,4,5) to generate a family of similar triangles. 	3 : 4 : 5 5 : 12 : 13 7 : 24 : 25 8 : 15 : 17 9 : 40 : 41 11 : 60 : 61 12 : 35 : 37 13 : 84 : 85 15 : 112 : 113 16 : 63 : 65 17 : 144 : 145 19 : 180 : 181 20 : 21 : 29 20 : 99 : 101 21 : 220 : 221 23 : 264 : 265	24 : 143 : 145 28 : 45 : 53 28 : 195 : 197 32 : 255 : 257 33 : 56 : 65 36 : 77 : 85 39 : 80 : 89 44 : 117 : 125 48 : 55 : 73 51 : 140 : 149 52 : 165 : 173 57 : 176 : 185 60 : 91 : 109 60 : 221 : 229 65 : 72 : 97 68 : 285 : 293	69 : 260 : 269 84 : 187 : 205 85 : 132 : 157 88 : 105 : 137 95 : 168 : 193 96 : 247 : 265 104 : 153 : 185 105 : 208 : 233 115 : 252 : 277 119 : 120 : 169 120 : 209 : 241 133 : 156 : 205 140 : 171 : 221 160 : 231 : 281 161 : 240 : 289 204 : 253 : 325