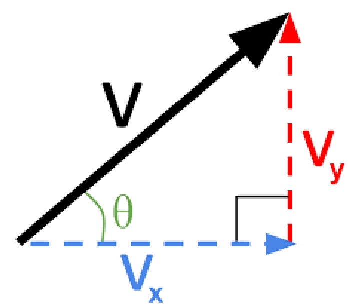
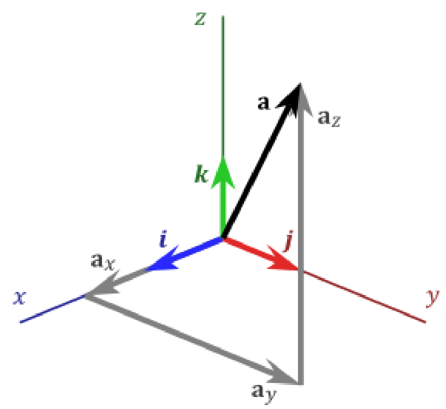
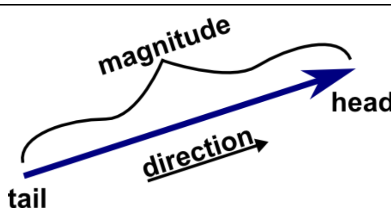
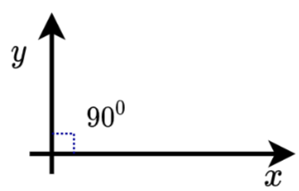
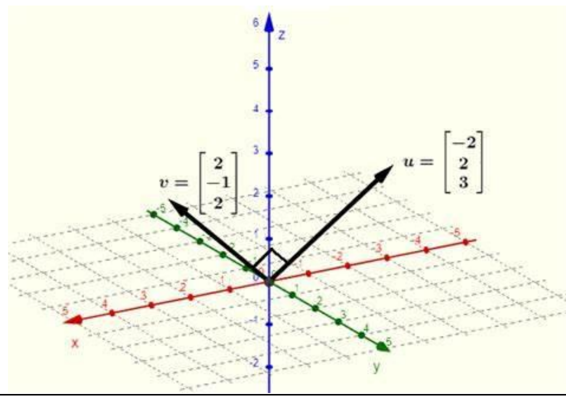
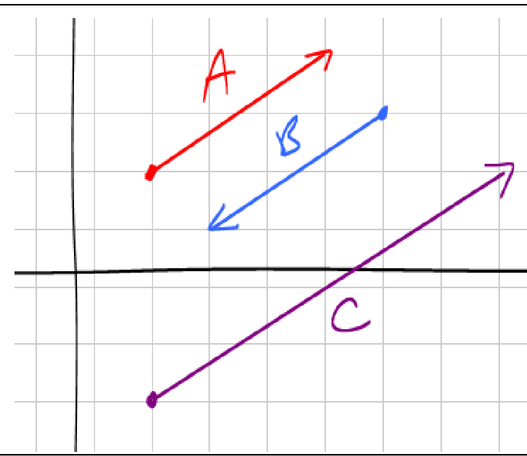
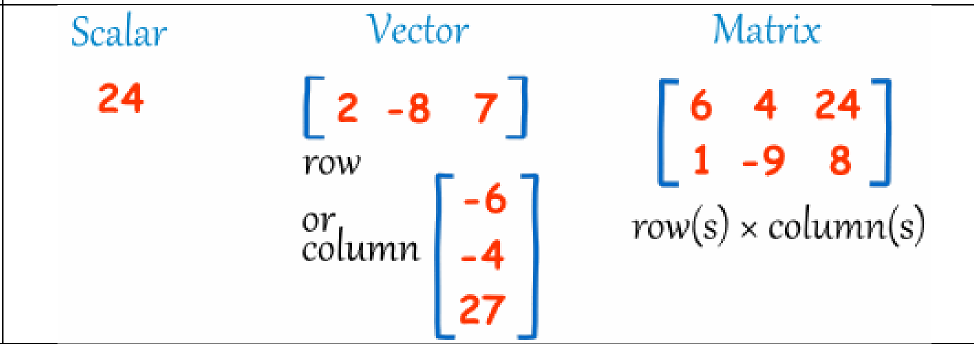
Harold's Vectors Cheat Sheet

20 January 2026

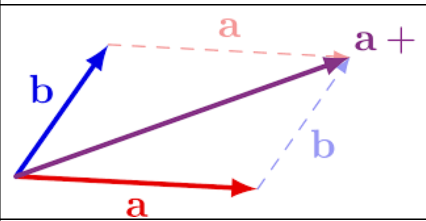
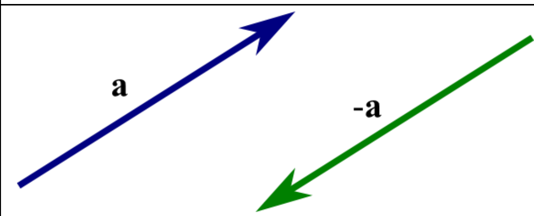
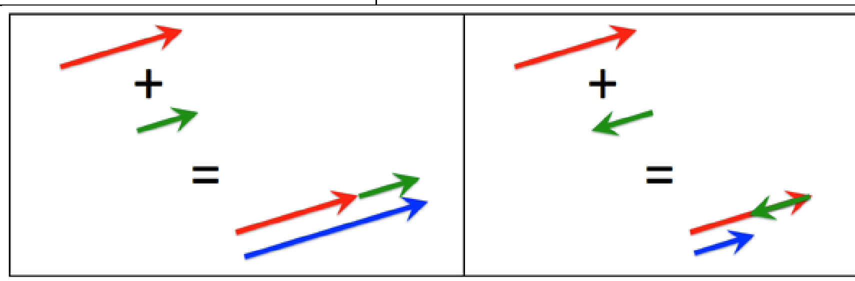
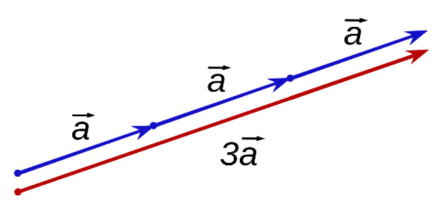
Definitions

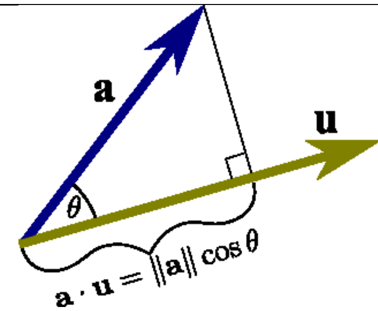
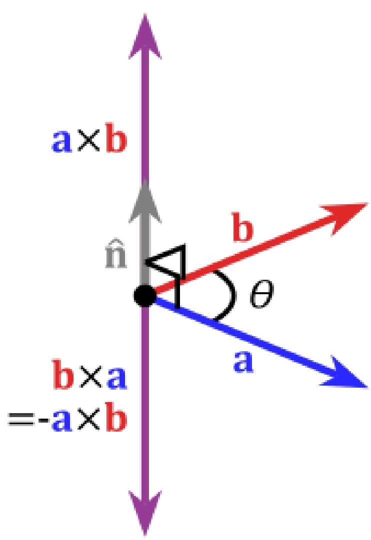
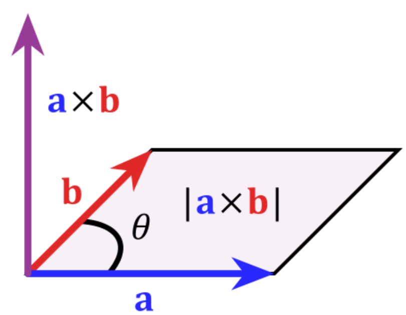
Term	Formula	Example
Vector Notation	$\mathbf{A}, \boldsymbol{a}$	Bold letter
	$\vec{a}, \vec{a}, \vec{a}$	Arrow or harpoon on top
Component Notation	$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$	$\boldsymbol{a} = 3\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}} + 5\hat{\boldsymbol{k}}$
	$\boldsymbol{a} = \langle a_x, a_y, a_z \rangle$	$\boldsymbol{a} = \langle 3, 4, 5 \rangle$
	$\boldsymbol{a} = r\angle\theta$ (2D)	$\boldsymbol{a} = 5 \angle 53.13°$ (2D)
	 2D	 3D
2D	*Polar → Rectangular*	*Rectangular → Polar*
	$x = r\cos\theta$ $y = r\sin\theta$ $tan\,\theta = \left(\frac{y}{x}\right)$	$r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = tan^{-1}\left(\frac{y}{x}\right)$
Vectors Used in Examples	$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$ $\boldsymbol{b} = b_x\hat{\boldsymbol{\imath}} + b_y\hat{\boldsymbol{\jmath}} + b_z\hat{\boldsymbol{k}}$ **2D**: set $a_z = b_z = 0$	$\boldsymbol{a} = 3\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}} + 5\hat{\boldsymbol{k}}$ $\boldsymbol{b} = 6\hat{\boldsymbol{\imath}} - 7\hat{\boldsymbol{\jmath}} - 8\hat{\boldsymbol{k}}$
Magnitude	$\lVert\boldsymbol{a}\rVert = \sqrt{a_x^2 + a_y^2 + a_z^2}$	$\lVert\boldsymbol{a}\rVert = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$
	Can also use $\lvert\boldsymbol{a}\rvert$. Length of vector, but with no direction (scalar). Similar to a hypotenuse. Think multi-dimensional Pythagorean Theorem.	
Direction	Divided into dimensional components.	A scalar with a direction is a vector. Example: speed vs. velocity
	$\tan\theta = \frac{a_y}{a_x}$	$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) \cong 53.13°$

Unit Vector (Basis Vector)	$\hat{\boldsymbol{\imath}} = x\text{-}axis = \langle 1, 0, 0 \rangle$ $\hat{\boldsymbol{\jmath}} = y\text{-}axis = \langle 0, 1, 0 \rangle$ $\widehat{\boldsymbol{k}} = z\text{-}axis = \langle 0, 0, 1 \rangle$	Circumflex or "hat" on top. Indicates direction only. Always has a magnitude of one (1 or unit).
	$\vec{\boldsymbol{u}} = \dfrac{\boldsymbol{a}}{\lVert \boldsymbol{a} \rVert}$	$\vec{\boldsymbol{u}} = \dfrac{a_x \hat{\boldsymbol{\imath}} + a_y \hat{\boldsymbol{\jmath}} + a_z \widehat{\boldsymbol{k}}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$
Scalar	*k, m*	A number with no direction or units.
Orthogonal	A change in one dimension does not change in any of the values in the other dimensions.	2D: right angle **Rectangular Coordinates:** The x-axis, y-axis, and z-axis are orthogonal to each other. **Polar Coordinates:** The angle is orthogonal to the line segment length
	if $\boldsymbol{a} \bullet \boldsymbol{b} = 0$	Two vectors are orthogonal if their dot product is zero.
		
Parallel	If $\boldsymbol{a} = k\boldsymbol{b}$	Two vectors are parallel if they have the same direction.
	Collinear but in opposite directions	
Vector vs. Matrix	vector = $1 \times n$ or $n \times 1$ matrix	A matrix with only one (1) row or column.

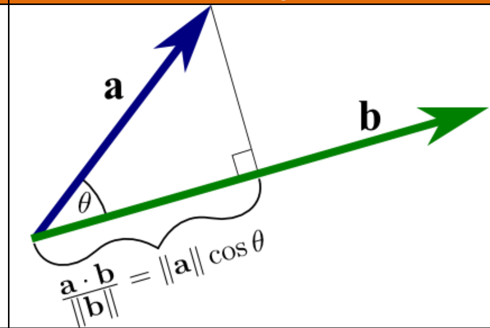
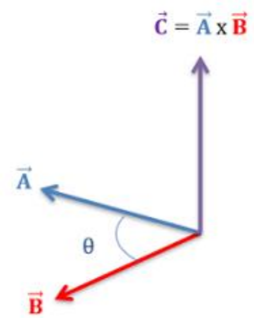
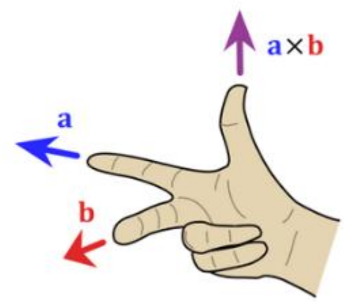
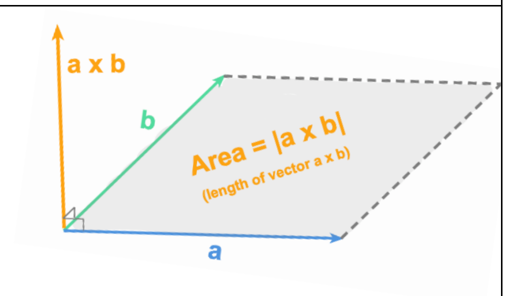
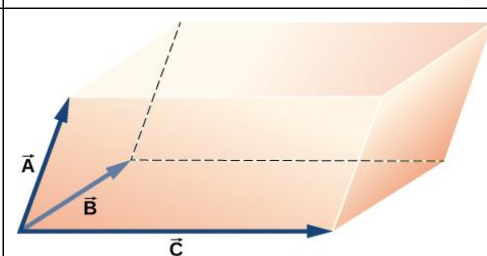
1D – 4D Classification	
Vector Types	

Vector Operations

Operation	Formula	Example
Addition	$\boldsymbol{a} + \boldsymbol{b} = (a_x + b_x)\hat{\boldsymbol{\imath}} + (a_y + b_y)\hat{\boldsymbol{\jmath}} + (a_z + b_z)\hat{\boldsymbol{k}}$	$\boldsymbol{a} + \boldsymbol{b} = (3+6)\hat{\boldsymbol{\imath}} + (4-7)\hat{\boldsymbol{\jmath}} + (5-8)\hat{\boldsymbol{k}} = 9\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}} - 3\hat{\boldsymbol{k}}$
	$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$	Commutative
	$(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c} = \boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c})$	Associative
	$(k + m)\boldsymbol{a} = k\boldsymbol{a} + m\boldsymbol{a}$	Distributive
	$k(\boldsymbol{a} + \boldsymbol{b}) = k\boldsymbol{a} + k\boldsymbol{b}$	
		
Subtraction	$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$	Change the direction of $\vec{b}$ then add.
		
Scalar Multiplication	$k\boldsymbol{a} = ka_x\hat{\boldsymbol{\imath}} + ka_y\hat{\boldsymbol{\jmath}} + ka_z\hat{\boldsymbol{k}}$	$3 \cdot \boldsymbol{a} = 3 \cdot 3\hat{\boldsymbol{\imath}} + 3 \cdot 4\hat{\boldsymbol{\jmath}} + 3 \cdot 5\hat{\boldsymbol{k}} = 9\hat{\boldsymbol{\imath}} + 12\hat{\boldsymbol{\jmath}} + 15\hat{\boldsymbol{k}}$
	Changes the magnitude only.	
	$k(\boldsymbol{a} + \boldsymbol{b}) = k\boldsymbol{a} + k\boldsymbol{b}$	
Dot Product (Scalar Product)	$\boldsymbol{a} \bullet \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$	$\boldsymbol{a} \bullet \boldsymbol{b} = (3)(6) + (4)(-7) + (5)(-8) = -50$
	$\boldsymbol{a} \bullet \boldsymbol{b} = \lVert\boldsymbol{a}\rVert\,\lVert\boldsymbol{b}\rVert \cos\theta$	$\cos\theta = \dfrac{\boldsymbol{a} \bullet \boldsymbol{b}}{\lVert\boldsymbol{a}\rVert\,\lVert\boldsymbol{b}\rVert}$
	$\boldsymbol{a} \bullet \boldsymbol{b} = \boldsymbol{b} \bullet \boldsymbol{a}$	Commutative
	$\boldsymbol{a} \bullet (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \bullet \boldsymbol{b} + \boldsymbol{a} \bullet \boldsymbol{c}$	Distributive
	$k(\boldsymbol{a} \bullet \boldsymbol{b}) = k\boldsymbol{a} \bullet \boldsymbol{b} = \boldsymbol{a} \bullet k\boldsymbol{b}$	Scalar Multiplication
	$0 \bullet \boldsymbol{u} = 0$	Zero Vector Dot Product
	$\boldsymbol{u} \bullet \boldsymbol{u} = \lVert\boldsymbol{u}\rVert^2$	Dot Product and Vector Magnitude Relationship
	Is always a scalar.	

		
Cross Product (Vector Product)	$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} \boldsymbol{i} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \boldsymbol{k}$ $= (a_y b_z - b_y a_z)\,\boldsymbol{i} - (a_x b_z - b_x a_z)\,\boldsymbol{j} + (a_x b_y - b_x a_y)\,\boldsymbol{k}$	
	$\lVert \boldsymbol{a} \times \boldsymbol{b} \rVert = \lVert \boldsymbol{a} \rVert \, \lVert \boldsymbol{b} \rVert \sin\theta$	$\sin\theta = \dfrac{\lVert \boldsymbol{a} \times \boldsymbol{b} \rVert}{\lVert \boldsymbol{a} \rVert \, \lVert \boldsymbol{b} \rVert}$
	$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a}$	Anti-Commutative
	$(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} \neq \boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$	Not Commutative
	$(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = (\boldsymbol{a} \bullet \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a} \bullet \boldsymbol{b})\,\boldsymbol{c}$	
	$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) \neq (\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}$	Not Associative
	$\boldsymbol{a} \bullet (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c}$	
	$\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c}$	Distributive
	$(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} \times \boldsymbol{c}$	
	$k(\boldsymbol{a} \times \boldsymbol{b}) = k\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{a} \times k\boldsymbol{b}$	Scalar Multiplication
	$(k\boldsymbol{a}) \times \boldsymbol{b} = k(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a} \times (k\boldsymbol{b})$	
	Is always a vector orthogonal to the other two vectors.	
		
Scalar Triple Product	$(\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c} = \boldsymbol{a} \bullet (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \bullet (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \bullet (\boldsymbol{a} \times \boldsymbol{b}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$	
Vector Triple Product	$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \bullet \boldsymbol{c})\,\boldsymbol{b} - (\boldsymbol{a} \bullet \boldsymbol{b})\,\boldsymbol{c}$	

Vector Applications

Application	Formula	Example
Projection	$proj_{\boldsymbol{a}}\boldsymbol{b} = \left(\frac{\boldsymbol{a} \cdot \boldsymbol{b}}{\boldsymbol{a} \cdot \boldsymbol{a}}\right)\boldsymbol{a}$	
Right Hand Rule	The cross product produces a vector orthogonal to the other two vectors.	Use the right hand rule to determine direction of the cross product vector.
		
Area (Parallelagram)	$A = \|\boldsymbol{a} \times \boldsymbol{b}\|$	
Volume (Parallelepiped)	$V = \|(\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c}\|$	
Torque	$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$	$\|\boldsymbol{\tau}\| = r\, F \sin\theta$
Coplanar	Three vectors are coplanar if $(\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$	All three vectors are in the same plane.